Homework #13

Due **Wednesday, October 26th** in Gradescope by 11:59 pm ET

Goal: Exploring Convergence of Infinite Series. Focus on Alternating Series Test, and Ratio Test. We will also focus on fluency of training, using multiple tests.

FIRST: Read through and understand the following Examples. Determine whether the given Series Converges Absolutely or Diverges. Justify.

Ex: $\displaystyle\sum_{n=1}^{\infty} \frac{n^n}{n! \cdot 2^n}$ Try Ratio Test:

$$L = \lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{\dfrac{(n+1)^{n+1}}{(n+1)!2^{n+1}}}{\dfrac{n^n}{n!\cdot 2^n}}\right| = \lim_{n\to\infty}\frac{(n+1)^{n+1}}{n^n}\cdot\frac{n!}{(n+1)!}\cdot\frac{2^n}{2^{n+1}}$$

$$= \lim_{n\to\infty}\frac{(n+1)^n(n+1)}{n^n}\cdot\frac{n!}{(n+1)n!}\cdot\frac{2^n}{2^n\cdot 2} = \lim_{n\to\infty}\left(\frac{n+1}{n}\right)^{n\to e}\cdot\left(\frac{1}{2}\right) = \frac{e}{2} > 1$$

The Original Series $\boxed{\text{Diverges by the Ratio Test}}$.

Ex: $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n(2n)!}{e^{2n}\cdot n!\cdot n^n}$ Try Ratio Test:

$$L = \lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{\dfrac{(-1)^{n+1}(2(n+1))!}{e^{2(n+1)}\,(n+1)!\,(n+1)^{n+1}}}{\dfrac{(-1)^n(2n)!}{e^{2n}\,n!\,n^n}}\right|$$

$$= \lim_{n\to\infty}\frac{(2n+2)!}{(2n)!}\cdot\frac{n^n}{(n+1)^{n+1}}\cdot\frac{e^{2n}}{e^{2n+2}}\cdot\frac{n!}{(n+1)!}$$

$$= \lim_{n\to\infty}\frac{(2n+2)(2n+1)(2n)!}{(2n)!}\cdot\frac{n^n}{(n+1)^n(n+1)}\cdot\frac{e^{2n}}{e^{2n}e^2}\cdot\frac{n!}{(n+1)n!}$$

$$= \lim_{n\to\infty}\frac{(2(n+1))(2n+1)}{(n+1)(n+1)}\cdot\left(\frac{n}{n+1}\right)^{n\to\frac{1}{e}}\cdot\frac{1}{e^2}$$

$$= \lim_{n\to\infty}\frac{2n+1}{n+1}\cdot\frac{\frac{1}{n}}{\frac{1}{n}}\cdot\left(\frac{2}{e^3}\right) = \lim_{n\to\infty}\frac{2+\frac{1}{n}\to 0}{1+\frac{1}{n}\to 0}\cdot\left(\frac{2}{e^3}\right) = 2\left(\frac{2}{e^3}\right) = \frac{4}{e^3} < 1$$

The original series is $\boxed{\text{Absolutely Convergent (A.C.) by the Ratio Test}}$.

Now complete the following HW problems

1. Consider $\sum_{n=1}^{\infty} \dfrac{n+1}{n^2 + 4n + 7}$. Use **two** Different methods, namely the Integral Test and the Limit Comparison Test, to prove that this series Diverges. *You can skip checking the Integral Test preconditions here this time. yay!*

2. Determine if the given Alternating series $\sum_{n=1}^{\infty} \dfrac{(-1)^{n+1}}{3n + 1}$ Converges or Diverges.

Determine if the given series is Absolutely Convergent or Divergent.

3. $\sum_{n=1}^{\infty} \dfrac{n}{5^n}$

4. $\sum_{n=1}^{\infty} \dfrac{(-3)^n}{(2n + 1)!}$

5. $\sum_{n=1}^{\infty} \dfrac{n!}{100^n}$

6. $\sum_{n=1}^{\infty} \dfrac{n!}{n^n}$

7. $\sum_{n=1}^{\infty} \dfrac{n^{100}\, 100^n}{n!}$

8. $\sum_{n=1}^{\infty} \dfrac{(2n)!}{(n!)^2}$

9. Consider the series $\sum_{n=1}^{\infty} \dfrac{\ln n}{n}$.

(a) Show that n^{th} Term Divergence Test is **Inconclusive**.

(b) Show that the Ratio Test is **Inconclusive**.

(c) Show that the series Diverges using the Integral Test. Skip checking the 3 preconditions here. **Note:** This is an example where the terms approach 0 but the series Diverges.

10. Prove that $\sum_{n=1}^{\infty} \dfrac{6}{n^6}$ is Convergent by using the Limit Comparison Test.

Note that this work will be a sample proof of the fact that *Constant multiple of a Convergent series* **is** *Convergent.*

11. Show that $\sum_{n=1}^{\infty} \dfrac{e^n}{n^2}$ Diverges using **two** Different methods.

REGULAR OFFICE HOURS

Monday: 12:00–3:00 pm

7:30–9:00 pm TA Aidee, SMUDD 206

9:00–10:30 pm TA Mia, SMUDD 206

Tuesday: 1:00–4:00 pm

6–7:30 pm TA Admire, SMUDD 206

7:30–9:00 pm TA Karime, SMUDD 206

9–10:30 pm TA Ali, SMUDD 206

Wednesday: 1:00-3:00 pm

6–7:30 pm TA Admire, SMUDD 206

7:30–9:00 pm TA Ali, SMUDD 206

Thursday: none for Professor

1-2:30 pm TA Mia, SMUDD 205

7:30–9:00 pm TA Aidee, SMUDD 206

9–10:30 pm TA Karime, SMUDD 207

Friday: 12:00–2:00 pm

Train your Convergence Tests Daily