

Homework #11

Due **Wednesday, October 19th** in Gradescope by 11:59 pm ET

Goal: Exploring Convergence of Infinite Series. Focus on Geometric Series and the n^{th} Term Divergence Test. We may also need L'Hôpital's Rule to finish some of the limits at hand.

FIRST: Read through and understand the following Examples. Determine whether the given Series Converges or Diverges. If it Converges, find the Sum value. Justify.

$$\text{Ex: } \sum_{n=1}^{\infty} \frac{(-1)^n 5^{n-1}}{3^{2n+1}} = -\frac{1}{3^3} + \frac{5}{3^5} - \frac{5^2}{3^7} + \frac{5^3}{3^9} + \dots \quad \text{Here } a = -\frac{1}{27} \text{ and } r = -\frac{5}{3^2} = -\frac{5}{9}.$$

Series **Converges by Geometric Series Test (GST)**, because $|r| = \left| -\frac{5}{9} \right| = \frac{5}{9} < 1$ with

$$\text{SUM} = \frac{a}{1-r} = \frac{-\frac{1}{27}}{1 - \left(-\frac{5}{9}\right)} = \frac{-\frac{1}{27}}{\frac{14}{9}} = -\frac{1}{27} \cdot \frac{9}{14} = -\frac{1}{3} \cdot \frac{1}{14} = \boxed{-\frac{1}{42}}$$

$$\text{Ex: } \sum_{n=0}^{\infty} \left(\frac{7}{3}\right)^n = 1 + \frac{7}{3} + \frac{7^2}{3^2} + \frac{7^3}{3^3} + \dots \quad \text{Here } a = 1 \text{ and } r = \frac{7}{3}.$$

Series **Diverges by GST**, because $|r| = \frac{7}{3} \geq 1$.

$$\text{Ex: } \sum_{n=1}^{\infty} \frac{e^n}{n^2} \quad \text{Diverges by the } n^{\text{th}} \text{ Term Divergence Test (nTDT) because}$$

$$\lim_{n \rightarrow \infty} \frac{e^n}{n^2} \stackrel{\infty}{=} \lim_{x \rightarrow \infty} \frac{e^x}{x^2} \stackrel{\text{L'H}}{=} \lim_{x \rightarrow \infty} \frac{e^x}{2x} \stackrel{\text{L'H}}{=} \lim_{x \rightarrow \infty} \frac{e^x}{2} = \infty \neq 0$$

$$\text{Ex: } \sum_{n=1}^{\infty} 3 \quad \text{Diverges by nTDT because } \lim_{n \rightarrow \infty} 3 = 3 \neq 0 \quad \text{Q: Is this also Geometric? } r = ?$$

$$\text{Ex: } \sum_{n=1}^{\infty} e^{\frac{1}{n}} \quad \text{Diverges by nTDT because } \lim_{n \rightarrow \infty} e^{\frac{1}{n}} = 1 \neq 0$$

Continue to NEXT Page for HW problems.

Determine whether each of the following Converge or Diverge. Justify.

1. $\{8\}_{n=1}^{\infty}$ 2. $\sum_{n=1}^{\infty} 8$ 3. $\left\{ \frac{2n}{3n+1} \right\}_{n=1}^{\infty}$ 4. $\sum_{n=1}^{\infty} \frac{2n}{3n+1}$

Determine whether the given series Converges or Diverges. If it converges, find the Sum value. Justify.

5. $\sum_{n=1}^{\infty} \frac{8}{5^n}$ 6. $\sum_{n=0}^{\infty} \frac{8}{5^n}$ 7. $\sum_{n=1}^{\infty} \frac{4^n}{9^{n-1}}$

8. $\sum_{n=1}^{\infty} \frac{7^{n+1}}{3^n}$ 9. $\sum_{n=1}^{\infty} (-1)^n \frac{4^{2n+1}}{3^{3n-1}}$ 10. $\sum_{n=1}^{\infty} e^n$

11. $\sum_{n=1}^{\infty} \frac{1+2^n}{3^n}$ 12. $\sum_{n=0}^{\infty} \frac{1}{(1999)^n}$ 13. $\sum_{n=1}^{\infty} \frac{1}{1999}$

14. $\sum_{n=1}^{\infty} \arctan n$ 15. $\sum_{n=2}^{\infty} \frac{n^2}{\ln n}$ 16. $\sum_{n=1}^{\infty} \sin^2 \left(\frac{\pi n^4 + 1}{3n^4 + 5} \right)$

17. $\sum_{n=1}^{\infty} \left(1 + \ln \left(1 + \frac{5}{n} \right) \right)^n$

Consider these variable versions of Geometric Series. Find the values of x for which the series Converges. Find the sum of the Series for those values of x (answer in terms of x).

18. $\sum_{n=1}^{\infty} (-5)^n x^n$ 19. $\sum_{n=0}^{\infty} \frac{(x-2)^n}{3^n}$

REGULAR OFFICE HOURS

Monday: 12:00–3:00 pm

7:30–9:00 pm TA Aidee, SMUDD 206

9:00–10:30 pm TA Mia, SMUDD 206

Tuesday: 1:00–4:00 pm

6–7:30 pm TA Admire, SMUDD 206

7:30–9:00 pm TA Karime, SMUDD 206

9–10:30 pm TA Ali, SMUDD 206

Wednesday: 1:00–3:00 pm

6–7:30 pm TA Admire, SMUDD 206

7:30–9:00 pm TA Ali, SMUDD 206

Thursday: none for Professor

1–2:30 pm TA Mia, SMUDD 205

7:30–9:00 pm TA Aidee, SMUDD 206

9–10:30 pm TA Karime, SMUDD 207

Friday: 12:00–2:00 pm

Challenge yourself to work differently this week...