Review Packet for Exam #3

Math 121-D. Benedetto

Interval of Convergence: Find the **Interval** and **Radius of Convergence** for each of the following power series. Analyze convergence at the endpoints carefully, with full justification.

1. $\displaystyle\sum_{n=1}^{\infty} \frac{(2x+3)^n}{n}$
2. $\displaystyle\sum_{n=1}^{\infty} \frac{(-3)^n x^n}{n^2 4^n}$
3. $\displaystyle\sum_{n=1}^{\infty} \frac{10^n (x+3)^n}{(n+1)^3 n!}$
4. $\displaystyle\sum_{n=0}^{\infty} \frac{2^n}{5n+1}(x+1)^n$
5. $\displaystyle\sum_{n=0}^{\infty} \frac{(n+2)!(x-5)^n}{10^n}$
6. $\displaystyle\sum_{n=0}^{\infty} \frac{\sqrt{n}(2x-1)^n}{4^n}$
7. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$
8. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n^n}$
9. $\displaystyle\sum_{n=1}^{\infty} \frac{(2n)!}{(3n)!} x^n$
10. $\displaystyle\sum_{n=2}^{\infty} \frac{\ln n}{n^2} x^n$
11. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n!}{n^3}(x-1)^n$
12. $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{n^{\frac{1}{2}}}$
13. $\displaystyle\sum_{n=1}^{\infty} (n+4)!\; n^n\; (x-3)^n$

Estimates: Use a Power Series Representation for each of the following functions to **estimate** each one within the given error. ESTIMATE ...

14. $\cos(1)$ with error less than $\dfrac{1}{100}$
15. $e^{-\frac{1}{3}}$ with error less than $\dfrac{1}{100}$
16. $\arctan 1$ with error less than .20
17. $\dfrac{1}{e}$ with error less than $\dfrac{1}{10}$
18. $\sin(1)$ with error less than $\dfrac{1}{100}$
19. $\dfrac{1}{\sqrt{e}}$ with error less than $\dfrac{1}{100}$
20. $\sin\left(\dfrac{1}{2}\right)$ with error less than $\dfrac{1}{100}$
21. $\arctan\left(\dfrac{1}{2}\right)$ with error less than $\dfrac{1}{100}$
22. $\ln 2$ with error less than $\dfrac{1}{5}$
23. $\cos\left(\dfrac{1}{2}\right)$ with error less than $\dfrac{1}{100}$
24. $\ln\left(\dfrac{3}{2}\right)$ with error less than $\dfrac{1}{10}$

MacLaurin Series: Find the MacLaurin Series for each of the following functions, and **state** the corresponding Radius of Convergence. Answer in Sigma notation.

25. $x^2 e^{-3x^4}$

26. $\dfrac{1-e^{-x}}{x}$

27. $x^4 \ln(1+x^3)$

28. $\dfrac{x^6}{1+7x}$

29. $x \arctan(2x)$

30. $\dfrac{d}{dx} x^5 \sin\left(x^3\right)$

31. $\displaystyle\int 3xe^{-3x^7}\ dx$

32. $\dfrac{d}{dx} x^4 \ln\left(1+8x\right)$

33. $\displaystyle\int 6x^3 \cos\left(6x^2\right)\ dx$

34. $\dfrac{1}{(1+7x)^2}$ Hint: $\dfrac{1}{(1+7x)^2} = \dfrac{d}{dx}\left(\dfrac{-1}{7(1+7x)}\right)$

Power Series Representations of Functions: Use a Power Series Representation for each of the following functions to compute the given integral. ESTIMATE each one within the given error.

35. $\displaystyle\int_0^1 x^2 \cos(x^3)\ dx$ with error less than $\dfrac{1}{50}$

36. $\displaystyle\int_0^{\frac{1}{2}} x \arctan x\ dx$ with error less than 0.01

37. $\displaystyle\int_0^1 \sin(x^2)\ dx$ with error less than 0.1

38. $\displaystyle\int_0^{\frac{1}{2}} e^{-x^3}\ dx$ with error less than 0.01

Sums: Find the **sum** for each of the following series.

39. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n 2^{n+2}}{3^n}$

40. $1+1+\dfrac{1}{2!}+\dfrac{1}{3!}+\dfrac{1}{4!}+\ldots$

41. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n}}{(2n)!}$

42. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n 49^n \pi^{2n}}{4^n (2n+1)!}$

43. $\displaystyle\sum_{n=0}^{\infty} \frac{(-9)^n \pi^{2n+1}}{4^n (2n)!}$

44. $\displaystyle\sum_{n=0}^{\infty} \frac{(-\pi^2)^n}{36^n (2n)!}$

45. $\displaystyle\sum_{n=0}^{\infty} \frac{x^{7n+1}}{n!}$

46. $-\dfrac{1}{3}+\dfrac{1}{5}-\dfrac{1}{7}+\dfrac{1}{9}-\ldots$

47. $1-\dfrac{1}{2}+\dfrac{1}{2^2\ 2!}-\dfrac{1}{2^3\ 3!}+\dfrac{1}{2^4\ 4!}+\ldots$

48. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{2^{n+1}(n+1)}$

49. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)3^n}$

50. $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{2n+1}$

Limits: Compute each of the following limits in two ways: first using L'H Rule and second using series.

51. $\lim_{x\to 0} \dfrac{\sin(3x) - 3x}{x - \arctan x}$

52. $\lim_{x\to 0} \dfrac{xe^x - \arctan x}{\ln(1+3x) - 3x}$

Sequence Limits: Use Series to show that

53. $\lim_{n\to\infty} \dfrac{6^n}{n!} = 0$

54. $\lim_{n\to\infty} \dfrac{n^n\ n!}{(3n)!} = 0$

Integrals: Use Series to compute

55. $\displaystyle\int \cos\left(x^2\right) - 1 + \frac{x^4}{2}\ dx$. Your answer should be in sigma notation $\displaystyle\sum_{n=2}^{\infty}$.

56. $\displaystyle\int \sin\left(x^2\right) - x^2\ dx$. Your answer should be in sigma notation $\displaystyle\sum_{n=1}^{\infty}$.

57. $\displaystyle\int 1 - \cos\left(x^2\right)\ dx$. Your answer should be in sigma notation $\displaystyle\sum_{n=1}^{\infty}$.

58. $\displaystyle\int 1 - x^2 - e^{-x^2}\ dx$. Your answer should be in sigma notation $\displaystyle\sum_{n=2}^{\infty}$.

59. $\displaystyle\int \arctan(2x) - 2x + \frac{8x^3}{3}\ dx$. Your answer should be in sigma notation $\displaystyle\sum_{n=2}^{\infty}$.

Derivations of MacLaurin Series: Solving for $+C$ is needed if using Integration

60. Prove the MacLaurin Series formula for arctan x.

61. Use two different methods to Prove the MacLaurin Series formula for $\ln(1+x)$.

62. Use two different methods to Prove the MacLaurin Series formula for $\cos x$.

63. Use three different methods to Prove the MacLaurin Series formula for $\sin x$.

64. Use two different methods to Prove the MacLaurin Series formula for $\ln(3+x)$.