Math 121 Midterm Exam #3 December 1, 2021

- This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or other aids are permitted.

- Numerical answers such as $\sin\left(\frac{\pi}{6}\right)$, $4^{\frac{3}{2}}$, $e^{\ln 4}$, $\ln(e^7)$, $e^{3\ln 3}$, $\arctan\sqrt{3}$ or $\cosh(\ln 3)$ should be simplified.

- Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

1. [26 Points] Analyze carefully and with full justification.

(a) Find the **Interval** and **Radius** of Convergence for $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n\,(6x-5)^n}{n^6 \cdot 7^n}$.

(b) Show that the MacLaurin Series for $\sin x$ has an *Infinite* Radius of Convergence.

(c) Design a Power Series which is convergent **only** at $x = 8$. Once you create your series, then proceed to justify that the Interval of Convergence is indeed $I = \{8\}$.

2. [14 Points] Use Series to compute each of the following. **State** the Radius of Convergence. Your answer should be in sigma notation $\displaystyle\sum_{n=0}^{\infty}$.

(a) $\dfrac{d}{dx}\left[7x^4 \arctan(7x)\right]$

(b) $\displaystyle\int \frac{x^3}{5+x}\,dx = \int x^3\left(\frac{1}{5+x}\right)\,dx$

3. [12 Points] Use Series to compute $\displaystyle\lim_{x\to 0} \frac{e^x - e^{-x} - 2x}{x^3}$.

Check your answer using L'Hôpital's Rule.

4. [10 Points] Use the Series to **Estimate** $\ln\left(\dfrac{3}{2}\right) = \ln\left(1+\dfrac{1}{2}\right)$ with error less than $\dfrac{1}{50}$. Justify.

5. [28 Points] Find the **sum** for each of the following convergent series. Simplify, if possible.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n \ \pi^{2n}}{9^{n-1}(2n+1)!}$

(b) $4 + 4 - \dfrac{4}{3} + \dfrac{4}{5} - \dfrac{4}{7} + \dfrac{4}{9} - \ldots$

(c) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n \ \left(\pi^2\right)^{n+1}}{\left(\sqrt{6}\right)^{4n} \ (2n)!} = \sum_{n=0}^{\infty} \frac{(-1)^n \ \pi^{2n+2}}{\left(\sqrt{6}\right)^{4n} \ (2n)!}$

(d) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^{n+1} \ (\ln 8)^n}{3^n \ n!}$

(e) $-1 - 1 + \dfrac{1}{2} - \dfrac{1}{3} + \dfrac{1}{4} - \dfrac{1}{5} + \ldots$

(f) $1 - \dfrac{1}{e} + \dfrac{1}{e^2 \ 2!} - \dfrac{1}{e^3 \ 3!} + \dfrac{1}{e^4 \ 4!} - \ldots$

6. [10 Points] Prove the MacLaurin Series formula for $\arctan x$. Yes, show that $C = 0$.

Answer should be in Sigma notation $\displaystyle\sum_{n=0}^{\infty}$

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OPTIONAL BONUS

Do not attempt these unless you are completely done with the rest of the exam.

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OPTIONAL BONUS #1 Compute $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n \ (3n+2)}{(n+1)(2n+1) \ 3^{n+1}}$