Math 121 Exam #2 Nov 2-6, 2022

Due Sunday, November 6, in Gradescope by 11:59 pm ET

- This is **NOT** an Open Notes Exam. You can **NOT** access any materials, homeworks problems, lecture notes, etc. You may use one 5x7 Cheat Sheet.
- There is **NO** *Open Internet* access allowed. Do **NOT** use any online sources.
- You are not allowed to discuss these problems with anyone, including Math Fellows.
- Submit your final work in Gradescope in the Exam 2 entry.
- Please *show* all of your work and *justify* all of your answers. No Calculators.

1. [60 Points] Compute the following **Improper** integrals. Simplify all answers. Justify.

(a) $\displaystyle\int_0^e x \ln x \, dx = \boxed{\frac{e^2}{4}}$

(b) $\displaystyle\int_0^{\frac{1}{2}} \frac{1}{x \ln x} \, dx = \boxed{-\infty}$

(c) $\displaystyle\int_3^{\infty} \frac{20 - x}{x^2 - 4x + 7} \, dx = \boxed{-\infty}$

(d) $\displaystyle\int_{-4}^{3} \frac{20 - x}{x^2 - 4x - 32} \, dx = \boxed{-\infty}$

(e) $\displaystyle\int_0^1 \frac{e^{\frac{1}{x}}}{x^2} \, dx = \boxed{\infty}$

(f) $\displaystyle\int_1^{\infty} \frac{e^{\frac{1}{x}}}{x^2} \, dx = \boxed{e - 1}$

(g) $\displaystyle\int_0^{e^3} \frac{1}{x\left[9 + (\ln x)^2\right]} \, dx = \boxed{\frac{\pi}{4}}$

2. [10 Points] Consider the series $\displaystyle\sum_{n=2}^{\infty} \frac{e^n}{\ln n}$. Demonstrate **Two Different** methods to show that this series **Diverges**.

3. [8 Points] Use the Absolute Convergence Test to show that $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{\cos^2 n}{n^6 + 7}$ **Converges**.

4. [30 Points] Determine whether each of the given series **Converges** or **Diverges**. Name any convergence test(s) you use, and justify all of your work.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{n^6+7}{7n^6+6}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{6 \cdot n!}{7 \cdot\ n^n}$ (c) $\displaystyle\sum_{n=1}^{\infty} \frac{n^6+7}{n^7}$

(d) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{n^7+6} + \frac{6^n}{7^n} \right)$ (e) $\displaystyle\sum_{n=1}^{\infty} \left(1 - \frac{7}{n^6}\right)^{n^6}$

5. [8 Points] Use the Integral Test to determine if $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n \cdot (\ln n)^2}$ **Converges** or **Diverges**.

Note: You do **not** have to check the 3 pre-conditions.

6. [24 Points] Determine whether the given series is **Absolutely Convergent**, **Conditionally Convergent**, or **Divergent**. Name any convergence test(s) you use, and justify all of your work.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{7n+6}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \cdot n^n \cdot (2n)!}{n^6 \cdot 6^n \cdot (n!)^3}$

(c) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n^2+6n+7}{n^6+7n+6}$