

Review Packet for Exam #1

Fall 2021, Math 121, Professor Danielle Benedetto

Derivatives: Compute the derivative for each of the following functions. Do not worry about simplifying your answers:

1. $f(x) = \arcsin x \cdot \arctan x + \arctan(\sin(\ln x))$

2. $f(x) = \frac{\sinh(x^2 - 2)}{x + \sin^{-1} x}$

3. $f(x) = \sin(e^{\arcsin e^x})$

4. $f(x) = \ln\left(1 - \arcsin\left(\frac{7}{x}\right)\right)$

5. Compute $f''(x)$ where $f(x) = \ln(1 - x)$.

6. Compute $f''(x)$ where $f(x) = \arctan(3x)$.

7. Compute $f''(x)$ where $f(x) = \arcsin(4x)$.

Proofs:

8. Prove that $\frac{d}{dx} \arcsin x = \frac{1}{\sqrt{1-x^2}}$

9. Prove that $\frac{d}{dx} \arctan x = \frac{1}{1+x^2}$

10. ~~Prove that $\frac{d}{dx} \arcsin x = \frac{1}{\sqrt{1-x^2}}$~~

11. Prove that $\frac{d}{dx} \arcsin(5x) = \frac{5}{\sqrt{1-25x^2}}$

12. Prove that $\frac{d}{dx} \ln x = \frac{1}{x}$

13. Prove that $\frac{d}{dx} \arctan(3x) = \frac{3}{1+9x^2}$

14. Prove, using Integration, that $\int \frac{1}{9+x^2} dx = \frac{1}{3} \arctan\left(\frac{x}{3}\right) + C$

15. Prove, using Integration, that $\int \frac{1}{\sqrt{16-x^2}} dx = \arcsin\left(\frac{x}{4}\right) + C$

Limits: Compute each of the following limit.

16. $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x$

17. $\lim_{x \rightarrow 0} \frac{7xe^x - \arctan(7x)}{\sin x + \ln(1-x)}$

18. $\lim_{x \rightarrow 0} \frac{\cos(4x) - 1 - \arctan(4x) + 4x}{\ln(1-x) + \arcsin x}$

19. $\lim_{x \rightarrow 0} \frac{1 - e^{-4x} - \arctan(4x)}{x^2}$

20. $\lim_{x \rightarrow 0^+} x^3 \ln x$

21. $\lim_{x \rightarrow 0^+} x \ln \left(\frac{1}{x}\right)$

22. $\lim_{x \rightarrow \infty} x^{\frac{1}{x^2}}$

23. $\lim_{x \rightarrow 0^+} (1 - 2x)^{\frac{1}{x}}$

24. $\lim_{x \rightarrow \infty} (x^3 + 1)^{\frac{1}{\ln x}}$

25. $\lim_{x \rightarrow \infty} x^2 \sin \left(\frac{1}{x^2}\right)$

26. $\lim_{x \rightarrow 0^+} \sqrt{x} \ln x$

27. $\lim_{x \rightarrow \infty} \left(1 - \frac{3}{x}\right)^{4x}$

28. $\lim_{x \rightarrow \infty} \left(1 + \ln \left(1 - \frac{6}{x^2}\right)\right)^{x^2}$

29. $\lim_{x \rightarrow \infty} \left(1 - \arctan \left(\frac{5}{x^4}\right)\right)^{3x^4}$

30. CHALLENGE: $\lim_{x \rightarrow \infty} \left(\sqrt{1 - \frac{5}{x^2}} - \sin \left(\frac{1}{x^2}\right)\right)^{5x^2}$

31. $\lim_{x \rightarrow \infty} \left(e^{\frac{1}{x^8}} - \frac{8}{x^8}\right)^{x^8}$

32. $\lim_{x \rightarrow \infty} \left(\frac{x}{x+3}\right)^x$

Integrals: Compute each of the following integrals.

33. $\int (e^x + x)^2 dx$

34. $\int x \sin^2 x dx$

35. $\int \frac{1}{\sqrt{25 - x^2}} dx$

36. $\int \frac{1}{x^2 + 25} dx$

37. $\int \frac{1}{x\sqrt{9 - \ln^2 x}} dx$

38. $\int x \arcsin x dx$

39. $\int \frac{1}{(4 - x^2)^{\frac{3}{2}}} dx$

40. $\int_1^e \ln x dx$

41. $\int \frac{\ln(2x^5)}{x^2} dx$

42. $\int \cos^5 x dx$

43. $\int \ln^2(x^{20}) dx$

44. $\int \sin^5 x \cos^2 x dx$

45. $\int \sin^2 x \cos^3 x dx$

46. $\int_{e^{\sqrt{3}}}^{e^3} \frac{1}{x(9 + (\ln x)^2)} dx$

47. $\int_1^e \frac{1}{x(1 + (\ln x)^2)^{\frac{3}{2}}} dx$

48. $\int_0^{\frac{\pi}{2}} \frac{\cos x}{(1 + \sin^2 x)^{\frac{7}{2}}} dx$

49. $\int_1^{\sqrt{3}} \frac{1}{\sqrt{4 - x^2}} dx$

50. $\int \frac{1}{(x^2 + 4)^2} dx$

51. $\int \frac{1}{(x^2 + 4)^{\frac{7}{2}}} dx$
52. $\int x^4 \arcsin x dx$
53. $\int x \arctan x dx$
54. $\int_0^1 x \tan^{-1}(x^2) dx$
55. $\int \frac{x^2}{x^6 + 1} dx$
56. $\int_1^{e^2} x \ln \sqrt{x} dx$
57. $\int \frac{x^2}{(1 - x^2)^{\frac{3}{2}}} dx$
58. $\int_1^e (\ln x)^2 dx$
59. $\int_0^{\sqrt{3}} \frac{1}{\sqrt{4 - x^2}} + \frac{1}{x^2 + 9} dx$
60. $\int \frac{x^2}{\sqrt{16 - x^2}} dx$
61. $\int x^3 \sqrt{9 - x^2} dx$
62. $\int \frac{x^2}{x^2 + 3} dx$
63. $\int_{-3}^3 \sqrt{9 - x^2} dx$
64. $\int_1^e \sqrt{x} \ln x dx$
65. $\int \frac{x + 3}{\sqrt{4 - x^2}} dx$
66. CHALLENGE: $\int (\arcsin x)^2 dx$