Quiz #7 Final Answers

1(a). $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n (3x+1)^n}{(n+7)\,7^n}$

Ratio Test

$$L = \lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{\dfrac{\cancel{(-1)^{n+1}}(3x+1)^{n+1}}{(n+8)\,7^{n+1}}}{\dfrac{\cancel{(-1)^n}(3x+1)^n}{(n+7)\,7^n}}\right| = \lim_{n\to\infty}\underbrace{\left|\frac{(3x+1)^{n+1}}{(3x+1)^n}\right|}_{|3x+1|}\cdot\underbrace{\frac{n+7}{n+8}}_{\to 1}\cdot\frac{\cancel{7^n}}{\underbrace{7^{n+1}}_{\cancel{7^n}\cdot 7}}$$

$$= \lim_{n\to\infty}\frac{|3x+1|}{7} = \frac{|3x+1|}{7} < 1$$

Converges by Ratio Test when

$|3x+1| < 7$ $\quad -7 < 3x+1 < 7$

$\quad\quad -1 \quad -1 \quad -1$

$-8 < 3x < 6$

$-8/3 < x < 2$

Manually Test Convergence at Endpoints

Take $x = -8/3$. Series becomes

$$\sum_{n=0}^{\infty}\frac{(-1)^n\left[3\left(-\frac{8}{3}\right)+1\right]^n}{(n+7)\cdot 7^n} = \sum_{n=1}^{\infty}\frac{\overbrace{(-1)^n(-7)^n}^{(*)\ (-1)^n\,\cancel{7^n}}}{(n+7)\,\cancel{7^n}} = \sum_{n=1}^{\infty}\frac{\overbrace{\cancel{(-1)^{2n}}}^{\text{Even}\ \to 1}}{n+7} = \sum_{n=1}^{\infty}\frac{1}{n+7} \approx \sum_{n=1}^{\infty}\frac{1}{n}$$

Diverges p-Series $p=1$.

(*) OR $(-1)^n(-7)^n = [(-1)(-7)]^n = 7^n$

OR $(-1)^n(-1)^n 7^n = [(-1)(-1)]^n 7^n = \cancel{1^n}\,7^n$

$$\lim_{n\to\infty}\frac{\frac{1}{n+7}}{\frac{1}{n}} = \lim_{n\to\infty}\frac{\overbrace{n}^{\to 1}}{n+7} = 1 \quad \text{Finite, Non-Zero}$$

$\Rightarrow$ Series Diverges by LCT

Take $x = 2$. Series becomes

$$\sum_{n=1}^{\infty}\frac{(-1)^n\overbrace{(3(2)+1)^n}^{7^n}}{(n+7)\,\cancel{7^n}} = \sum_{n=0}^{\infty}\frac{(-1)^n}{n+7}$$

Converges by AST

① $b_n = \dfrac{1}{n+7} > 0$

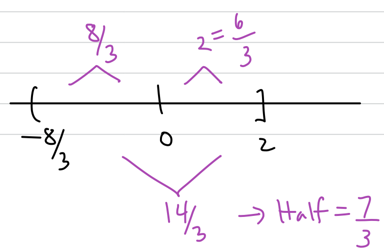
② $\displaystyle\lim_{n\to\infty} b_n = \lim_{n\to\infty}\frac{\overbrace{1}^{0}}{\underbrace{n+7}_{\infty}} = 0$

③ Terms decreasing

$b_{n+1} = \dfrac{1}{n+8} \le \dfrac{1}{n+7} = b_n$

Finally, $I = \left(-8/3,\, 2\right]$

$R = \dfrac{7}{3}$



1(b) $\sum_{n=1}^{\infty} n^n (x-6)^n$

Ratio Test

$$L = \lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{\overbrace{(n+1)^{n+1}}^{(n+1)^n\cdot(n+1)}(x-6)^{n+1}}{n^n(x-6)^n}\right| = \lim_{n\to\infty}\underbrace{\frac{(n+1)^n}{n^n}}_{\to e}\cdot\underbrace{(n+1)}_{\to\infty}|x-6| = \infty > 1$$

Diverges by Ratio Test for all x unless x=6

when L = 0<1

Finally, $I = \{6\}$

$R = 0$

1(c) $\sum_{n=1}^{\infty} \frac{x^{n+1}}{n!}$

Ratio Test

$$L = \lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{\frac{x^{n+2}}{(n+1)!}}{\frac{x^{n+1}}{n!}}\right| = \lim_{n\to\infty}\left|\frac{\overbrace{x^{n+2}}^{|x|}}{x^{n+1}}\right|\cdot\frac{\cancel{n!}}{\underbrace{(n+1)!}_{(n+1)\cancel{n!}}} = \lim_{n\to\infty}\frac{|x|}{\underbrace{n+1}_{\infty}} \to 0 = 0 < 1$$

Always

Converges by Ratio Test for all x in $\mathbb{R}$

Finally, $I = (-\infty, \infty)$

$R = \infty$