Quiz #6 Final Answers

Fall 2021

1. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{4+7^n} \xrightarrow[\text{A.S.}]{\text{Must}} \sum_{n=1}^{\infty} \frac{1}{4+7^n} \approx \sum_{n=1}^{\infty} \frac{1}{7^n}$ Converges by GST $|r| = \left|\frac{1}{7}\right| = \frac{1}{7} < 1$

Bound Terms

$$\frac{1}{4+7^n} \leq \frac{1}{7^n}$$

$\Rightarrow$ Absolute Series also Converges by C.T

$\Downarrow$

Original Series Converges by the Absolute Convergence Test

Note: o.s. is also A.C. by definition but that was not needed

2. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n 7^n \cdot n^n \cdot n!}{(2n)!\, n^7}$

Ratio Test

$$L = \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left| \frac{\dfrac{\cancel{(-1)^{n+1}}\, 7^{n+1} (n+1)^{n+1} (n+1)!}{[2(n+1)]!\, (n+1)^7}}{\dfrac{\cancel{(-1)^n}\, 7^n\, n^n\, n!}{(2n)!\, n^7}} \right|$$

$$= \lim_{n\to\infty} \frac{\overset{7}{\cancel{7^{n+1}}}}{\cancel{7^n}} \cdot \frac{\overset{(n+1)^n (n+1)}{(n+1)^{n+1}}}{n^n} \cdot \frac{\overset{(n+1)\cancel{n!}}{(n+1)!}}{\cancel{n!}} \cdot \frac{\cancel{(2n)!}}{\underset{(2n+2)(2n+1)\cancel{(2n)!}}{(2n+2)!}} \cdot \frac{n^7}{(n+1)^7}$$

$$= \lim_{n\to\infty} 7 \overset{e}{\cancel{\frac{(n+1)^n}{n^n}}} \cdot \frac{\cancel{(n+1)}(n+1)}{\underset{2\cancel{(n+1)}}{(2n+2)}(2n+1)} \frac{1/n}{1/n} \left(\frac{n}{n+1} \frac{1/n}{1/n}\right)^7$$

$$= \lim_{n\to\infty} \frac{7e}{2} \left(\frac{1 + \cancel{1/n}^{\,0}}{2 + \cancel{1/n}^{\,0}}\right)_{\frac{1}{2}} \left(\frac{1}{1 + \cancel{1/n}_{\,0}}\right)^{7}_{1} = \frac{7e}{4} > 1 \Rightarrow$$ Original Series Diverges by Ratio Test

O.S. A.S.

3. $\sum_{n=1}^{\infty} (-1)^n \frac{n^7+9}{n^9+7} \xrightarrow[\text{A.S.}]{\text{Must}} \sum_{n=1}^{\infty} \frac{n^7+9}{n^9+7} \approx \sum_{n=1}^{\infty} \frac{n^7}{n^9} = \sum_{n=1}^{\infty} \frac{1}{n^2}$

Converges p-Series

$p = 2 > 1$

LCT Limit

$$\lim_{n\to\infty} \frac{\frac{n^7+9}{n^9+7}}{\frac{1}{n^2}} = \lim_{n\to\infty} \frac{n^9+9n^2}{n^9+7} \cdot \frac{1/n^9}{1/n^9} = \lim_{n\to\infty} \frac{1+\cancel{9/n^7}^{\,0}}{1+\cancel{7/n^9}^{\,0}} = 1$$

Finite Non-Zero

note: AST not needed for O.S.

$\Rightarrow$ Absolute Series also Converges by LCT

$\Rightarrow$ Original Series Absolutely Convergent by Definition

O.S. A.S.

4. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{7n+2} \xrightarrow[\text{A.S.}]{\text{Must 1st}} \sum_{n=1}^{\infty} \frac{1}{7n+2} \approx \sum_{n=1}^{\infty} \frac{1}{n}$ Diverges, Harmonic, p-Series $p=1$

2nd

LCT Limit

$$\lim_{n\to\infty} \frac{\frac{1}{7n+2}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{n}{7n+2} \cdot \frac{1/n}{1/n} = \lim_{n\to\infty} \frac{1}{7+\cancel{2/n}^{\,0}} = \frac{1}{7}$$

Finite Non-Zero

$\Rightarrow$ Absolute Series also Diverges by LCT

AST on O.S.

① $b_n = \frac{1}{7n+2} > 0$ ✓

② $\lim_{n\to\infty} b_n = \lim_{n\to\infty} \frac{1^{\,\to 0}}{7n+2_{\,\to\infty}} = 0$ ✓

③ Terms Decreasing b/c ✓

$b_{n+1} = \frac{1}{\underbrace{7(n+1)+2}_{7n+9}} \le \frac{1}{7n+2} = b_n$

Original Series Converges by AST

$+$

extra:

OR Related Function $f(x) = \frac{1}{7x+2}$

has Derivative

$f'(x) = \frac{-7}{\underbrace{(7x+2)^2}_{+}} < 0$

Finally, the Original Series is Conditionally Convergent by Definition