Quiz 5 Final Answers — Fall 2021

1. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n\, 3^{n+1}}{2^{3n-1}} = \underset{n=1}{-\frac{3^2}{2^2}} + \underset{n=2}{\frac{3^3}{2^5}} - \underset{n=3}{\frac{3^4}{2^8}} + \cdots$

$a = \dfrac{-3^2}{2^2} = \dfrac{-9}{4}$

Converges by GST b/c $|r| = \left|-\frac{3}{8}\right| = \frac{3}{8} < 1$

$r = \dfrac{-3}{2^3} = \dfrac{-3}{8}$

with Sum $= \dfrac{a}{1-r} = \dfrac{-\frac{9}{4}}{1-\left(\frac{-3}{8}\right)} = \dfrac{-\frac{9}{4}}{\frac{11}{8}} = -\dfrac{9}{\cancel{4}} \cdot \dfrac{\cancel{8}^{\,2}}{11} = \dfrac{-18}{11}$ Match

2 $\displaystyle\sum_{n=1}^{\infty} \frac{4}{n^7} + \frac{4^n}{7^n} = \sum_{n=1}^{\infty} \frac{4}{n^7} + \sum_{n=1}^{\infty} \frac{4^n}{7^n}$ split

$\displaystyle = 4\sum_{n=1}^{\infty} \frac{1}{n^7} + \sum_{n=1}^{\infty} \left(\frac{4}{7}\right)^n$

Constant Multiple of Convergent p-Series $p = 4 > 1$ is Convergent

+

Convergent by GST with $|r| = \left|\frac{4}{7}\right| = \frac{4}{7} < 1$

Original Series Converges b/c the Sum of 2 Convergent Series is Convergent.

(OR by Arithmetic of Series)

3. $\displaystyle\sum_{n=1}^{\infty} \frac{n^7}{4\ln n}$ Diverges by nTDT since

$$\lim_{n\to\infty} \frac{n^7}{4\ln n} \;\frac{\infty}{\infty} = \lim_{x\to\infty} \frac{x^7}{4\ln x} \;\frac{\infty}{\infty} \underset{\text{L'H}}{=} \lim_{x\to\infty} \frac{7x^6}{4\cdot 1/x} \cdot \frac{x}{x} = \lim_{x\to\infty} \frac{7x^7}{4} \to \infty = \infty \neq 0$$

4. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^7+4} \approx \sum_{n=1}^{\infty} \frac{1}{n^7}$ Convergent p-Series $p=7>1$

Bound Terms

$\displaystyle\frac{1}{n^7+4} \leq \frac{1}{n^7}$ $\Rightarrow$ Original Series (also) Converges by C.T.

(note: OR LCT works too)

5. $\displaystyle\sum_{n=1}^{\infty} \frac{n^4+7}{n^7+4} \approx \sum_{n=1}^{\infty} \frac{n^4}{n^7} = \sum_{n=1}^{\infty} \frac{1}{n^3}$ Converges p-Series $p=3>1$

(note: not clear bounds b/c +7 bigger than +2
many terms $\rightsquigarrow$ LCT instead of CT)

LCT

$$\lim_{n\to\infty} \frac{\frac{n^4+7}{n^7+4}}{\frac{1}{n^3}} = \lim_{n\to\infty} \frac{n^7+7n^3}{n^7+4} \frac{1/n^7}{1/n^7} = \lim_{n\to\infty} \frac{1+\frac{7}{n^4}\to 0}{1+\frac{4}{n^7}\to 0} = 1$$

(Finite Non-Zero)

"Share Same Behavior"
"Comparable"

$\Rightarrow$ Original Series also Converges by LCT

6. $\displaystyle\sum_{n=1}^{\infty} \frac{n^3}{n^4+7} \approx \sum_{n=1}^{\infty} \frac{1}{n}$ Divergent p-Series $p=1$ (OR Harmonic)

(note: bound not in helpful order

$\displaystyle\frac{n^3}{n^4+7} \leq \frac{n^3}{n^4} = \frac{1}{n}$ "Smaller than Diverge is Inconclusive")

need LCT Limit

$$\lim_{n\to\infty} \frac{\frac{n^3}{n^4+7}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{n^4}{n^4+7} \frac{1/n^4}{1/n^4} = \lim_{n\to\infty} \frac{1}{1+\frac{7}{n^4}\to 0} = 1$$

(Finite Non-Zero)

"Share Same Behavior"
"Comparable"

$\Rightarrow$ Original Series also Diverges by LCT