Quiz #4 Final Answers — Fall 2021

1. $$\int_{-1}^{4} \frac{6}{x^2-2x-8}\,dx = \int_{-1}^{4} \frac{6}{(x-4)(x+2)}\,dx \overset{\text{PFD}}{=} \lim_{t\to 4^-} \int_{-1}^{t} \frac{1}{x-4} - \frac{1}{x+2}\,dx$$

Factor 1st

V.A. @ x=4

$$= \lim_{t\to 4^-} \ln|x-4| - \ln|x+2| \Big|_{-1}^{t}$$

$$= \lim_{t\to 4^-} \underbrace{\ln|\underbrace{t-4}_{0^+}|}_{-\infty} - \underbrace{\ln|t+2|}_{\ln 6 \text{ Finite}} - \big(\underbrace{\ln|-5|}_{\ln 5 \text{ Finite}} - \underbrace{\ln 1}_{0}\big)$$

10^{-1}

$= -\infty$ Diverges ✓

PFD

$$(x-4)(x+2)\left(\frac{6}{(x-4)(x+2)} = \frac{A}{x-4} + \frac{B}{x+2}\right)(x-4)(x+2)$$

$$6 = A(x+2) + B(x-4)$$
$$= Ax + 2A + Bx - 4B$$
$$= (A+B)x + 2A - 4B$$

Conditions

- $A+B=0 \Rightarrow B=-A$
- $2A-4B=6$ $2A-4(-A)=6$

$6A=6 \rightarrow A=1 \rightarrow B=-1$

2. $$\int_{-\infty}^{4} \frac{6}{x^2-2x+10}\,dx = \lim_{t\to-\infty} \int_{t}^{4} \frac{6}{x^2-2x+10}\,dx = \lim_{t\to-\infty} \int_{t}^{4} \frac{6}{\underbrace{(x-1)}_{u}^2+9}\,dx$$

Check: Does Not Factor

Discriminant

$b^2-4ac = 4-4(1)(10) = -36$

↳ Complete the Square

$$\underbrace{x^2-2x+1+9}_{10}$$

$$\boxed{\begin{array}{l} u=x-1 \\ du=dx \end{array}}$$

$$\boxed{\begin{array}{l} x=t \Rightarrow u=t-1 \\ x=4 \Rightarrow u=4-1=3 \end{array}}$$

$$= \lim_{t\to-\infty} \int_{t-1}^{3} \frac{6}{u^2+9}\,du$$

$$= \lim_{t\to-\infty} \overset{2}{\cancel{6}}\left(\frac{1}{\cancel{3}}\right)\arctan\left(\frac{u}{3}\right)\Big|_{t-1}^{3}$$

$$= \lim_{t\to-\infty} 2\left(\underbrace{\arctan\left(\tfrac{3}{3}\right)}_{\pi/4,\ 1} - \underbrace{\arctan\left(\tfrac{t-1}{3}\right)}_{-\pi/2,\ -\infty}\right)$$

$$= 2\left(\frac{\pi}{4} + \frac{\pi}{2}\right) = 2\left(\frac{3\pi}{4}\right) = \frac{3\pi}{2}$$ Converges ✓

3. $\displaystyle\int_0^1 x^3 \ln x\,dx = \lim_{t\to 0^+}\int_t^1 x^3 \ln x\,dx \overset{\text{IBP}}{=} \lim_{t\to 0^+} \frac{x^4}{4}\ln x\Big|_t^1 - \frac{1}{4}\int_t^1 x^3\,dx$

undefined @ $x=0$

IBP

$$\boxed{\begin{array}{ll} u=\ln x & dv = x^3\,dx \\ du=\frac{1}{x}dx & v=\frac{x^4}{4}\end{array}}$$

$$= \lim_{t\to 0^+} \frac{x^4}{4}\ln x\Big|_t^1 - \frac{x^4}{16}\Big|_t^1$$

$$= \lim_{t\to 0^+} \frac{1}{4}\overset{\to 0}{\cancel{\ln 1}} - \overset{0\cdot(-\infty)\ \to 0}{\cancel{\frac{t^4}{4}\ln t}} - \left(\frac{1}{16} - \overset{\to 0}{\cancel{\frac{t^4}{16}}}\right)$$

See (*) L'H

$$= -\frac{1}{16}$$

Converges ✓

$$(*)\ \lim_{t\to 0^+} \overset{0\cdot(-\infty)}{t^4\cdot \ln t} = \lim_{t\to 0^+} \frac{\ln t}{1/t^4}\ \frac{-\infty}{\infty} = \lim_{t\to 0^+}\frac{\frac{1}{t}}{\frac{-4}{t^5}} \quad \frac{-t^5}{4} \quad = \lim_{t\to 0^+} \frac{-\cancel{t^4}^{\to 0}}{4} = 0$$

$t^{-4} \to -4t^{-5}$