# Quiz #3 Final Answers   Fall 2021

$0 \qquad 0 \qquad \frac{0}{0}$

1. $$\lim_{x\to0} \frac{\ln(1-5x)+\arcsin(5x)}{3xe^x-\arctan(3x)}$$

$0 \qquad 0$

prep $5(1-25x^2)^{-1/2}$   $-5+5=\underline{0}$

prep $-5(1-5x)^{-1}$   $0+3-3=0$

$$\underset{L'H}{=}\lim_{x\to0}\frac{\frac{1}{1-5x}(-5)+\frac{1}{\sqrt{1-(5x)^2}}\cdot 5}{3xe^x+3e^x-\frac{1}{1+(3x)^2}\cdot 3}$$

prep $-3(1+9x^2)^{-1}$

$$\underset{L'H}{=}\lim_{x\to0}\frac{5(1-5x)^{-2}(-5)-\frac{5}{2}(1-25x^2)^{-3/2}(-50x)}{3xe^x+3e^x+3e^x\overset{--}{+}3(1+9x^2)^{-2}(18x)}$$

rewrite

$-25 \qquad 125x \qquad 0$

$$=\lim_{x\to0}\frac{\frac{-25}{(1-5x)^2}+\frac{\cancel{250}x}{\cancel{2}(1-25x^2)^{3/2}}}{3xe^x+6e^x+\frac{54x}{(1+9x^2)^2}}$$

$0 \qquad 6 \qquad 0$

$$=\boxed{-\frac{25}{6}}$$

Match!

$$2.\quad \lim_{x\to\infty}\left(1-\frac{8}{x^3}\right)^{x^3} \quad (1^\infty) = e^{\lim_{x\to\infty} \ln\left[\left(1-\frac{8}{x^3}\right)^{x^3}\right]}$$

$$= e^{\lim_{x\to\infty} x^3 \ln\left(1-\frac{8}{x^3}\right)} \quad (\infty\cdot 0)$$

FLIP

scratch

$$-8x^{-3} \to 24x^{-4}$$

$$= e^{\lim_{x\to\infty} \frac{\ln\left(1-\frac{8}{x^3}\right)}{1/x^3}} \quad 0/0$$

$$1/x^3 \to x^{-3} \to -3x^{-4}$$

L'H

$$= e^{\lim_{x\to\infty} \frac{\frac{1}{1-8/x^3}\left(\frac{24}{\cancel{x^4}}\right)}{\frac{-3}{x^4}}} \quad \left(\frac{-\cancel{x^4}}{3}\right)$$

FLIP BACK

ALGEBRA

don't drop

$$= e^{\lim_{x\to\infty} \cancel{\frac{1}{1-8/x^3}}^{\,1} \cdot (-8)}$$

$$= \boxed{e^{-8}}$$

Match!

3. $$\int_0^1 (x+1)\arctan x\,dx = \left(\frac{x^2}{2}+x\right)\arctan x\Big|_0^1 - \int_0^1 \frac{\frac{x^2}{2}+x}{1+x^2}\,dx$$

distribute

split-split

IBP

$$\boxed{\begin{array}{ll} u=\arctan x & dv = x+1\,dx \\ du = \dfrac{1}{1+x^2} & v=\dfrac{x^2}{2}+x \end{array}}$$

$$=\left(\frac{x^2}{2}+x\right)\arctan x\Big|_0^1 - \frac{1}{2}\int_0^1 \frac{x^2{+1-1}}{1+x^2}\,dx - \int_0^1 \frac{x}{1+x^2}\,dx$$

slip-in | slip-out  u-sub.

$$=\left(\frac{x^2}{2}+x\right)\arctan x\Big|_0^1 - \frac{1}{2}\left[\int_0^1 \cancel{\frac{x^2+1}{1+x^2}}^{\,1} - \frac{1}{1+x^2}\,dx\right] - \frac{1}{2}\int_1^2 \frac{1}{u}\,du$$

$$\boxed{\begin{array}{l} u=1+x^2 \\ du = 2x\,dx \\ \frac{1}{2}du = x\,dx \end{array}}$$

$$\boxed{\begin{array}{l} x=0 \Rightarrow u=1 \\ x=1 \Rightarrow u=2 \end{array}}$$

$$=\left(\frac{x^2}{2}+x\right)\arctan x\Big|_0^1 - \frac{1}{2}\left[x-\arctan x\right]\Big|_0^1 - \frac{1}{2}\ln|u|\,\Big|_1^2$$

$$=\frac{3}{2}\underbrace{\arctan 1}_{\pi/4} - 0 - \frac{1}{2}\left[1-\underbrace{\arctan 1}_{\pi/4} - (0-0)\right] - \frac{1}{2}\left(\ln 2 - \underbrace{\ln 1}_{0}\right)$$

$$=\frac{3\pi}{8} - \frac{1}{2} + \frac{\pi}{8} - \frac{\ln 2}{2}$$

$$=\underbrace{\frac{4\pi}{8}}_{\pi/2} - \frac{1}{2} - \frac{\ln 2}{2} = \boxed{\frac{\pi - 1 - \ln 2}{2}}$$

Match!

OR: You can split integral into 2 separate pieces.

$$\int_0^1 (x+1)\arctan x\,dx = \int_0^1 \underbrace{x\arctan x}_{\text{IBP 1}} + \underbrace{\arctan x\,dx}_{\text{IBP 2}} \quad \ldots$$

Optional Bonus:

pull in 3

find match?

$$\int \ln(3x^2+4) \cdot 1\, dx = x\ln(3x^2+4) - 6\int \frac{x^2}{3x^2+4}\,dx$$

$2 \cdot 3$

$u = \ln(3x^2+4) \quad dv = 1\,dx$

$du = \frac{1}{3x^2+4}(6x) \quad v = x$

$$= x\ln(3x^2+4) - 2\int \frac{3x^2+4-4}{3x^2+4}\,dx$$

Keep pulling out constants

Slip-in | Slip-out

split-split

$$= x\ln(3x^2+4) - 2\left[\int \cancel{\frac{3x^2+4}{3x^2+4}}^{\,1} dx - 4\int \frac{1}{3x^2+4}\,dx\right]$$

$$= x\ln(3x^2+4) - 2\left[x - 4\int \frac{1}{(\sqrt{3}x)^2+4}\,dx\right]$$

u-sub here

prep for "a"-rule

$u = \sqrt{3}x$

$du = \sqrt{3}\,dx$

$\frac{1}{\sqrt{3}}du = dx$

$$= x\ln(3x^2+4) - 2x + \frac{8}{\sqrt{3}}\int \frac{1}{u^2+4}\,du$$

now ready for "a-rule"

$$= x\ln(3x^2+4) - 2x + \frac{\cancel{8}^{\,4}}{\sqrt{3}} \cdot \frac{1}{\cancel{2}} \arctan\left(\frac{u}{2}\right) + C$$

$$\boxed{= x\ln(3x^2+4) - 2x + \frac{4}{\sqrt{3}}\arctan\left(\frac{\sqrt{3}x}{2}\right) + C}$$