Take Home Quiz #2, Fall 2021

1. $\int_2^{2\sqrt{3}} \frac{1}{\sqrt{16-x^2}} + \frac{1}{4+x^2}\,dx = \arcsin\left(\frac{x}{4}\right) + \frac{1}{2}\arctan\left(\frac{x}{2}\right)\Big|_2^{2\sqrt{3}}$ a-rules

$= \arcsin\left(\frac{2\sqrt{3}}{4}\right) + \frac{1}{2}\arctan\left(\frac{2\sqrt{3}}{2}\right) - \left(\arcsin\left(\frac{2}{4}\right) + \frac{1}{2}\arctan\left(\frac{2}{2}\right)\right)$

$= \arcsin\left(\frac{\sqrt{3}}{2}\right) + \frac{1}{2}\arctan\sqrt{3} - \arcsin\left(\frac{1}{2}\right) - \frac{1}{2}\arctan(1)$

$= \pi/3 + \pi/6 - \pi/6 - \pi/8$

$= \pi/3 - \pi/8 = \frac{8\pi}{24} - \frac{3\pi}{24} = \frac{5\pi}{24}$ Match ✓

2. $\int_{-\ln 2}^{-\ln\left(\frac{2}{\sqrt{3}}\right)} \frac{e^x}{\sqrt{1-e^{2x}}}\,dx = \int_{1/2}^{\sqrt{3}/2} \frac{1}{\sqrt{1-u^2}}\,du = \arcsin u\Big|_{1/2}^{\sqrt{3}/2}$

$= \arcsin\left(\frac{\sqrt{3}}{2}\right) - \arcsin\left(\frac{1}{2}\right)$

$= \pi/3 - \pi/6 = \frac{2\pi}{6} - \frac{\pi}{6} = \frac{\pi}{6}$ Match ✓

$(e^x)^2$

$u = e^x$

$du = e^x\,dx$

$x = -\ln 2 \Rightarrow u = e^{-\ln 2} = e^{\ln(2^{-1})} = \frac{1}{2}$

$x = -\ln\left(\frac{2}{\sqrt{3}}\right) \Rightarrow u = e^{-\ln\left(\frac{2}{\sqrt{3}}\right)} = e^{\ln\left[\left(\frac{2}{\sqrt{3}}\right)^{-1}\right]} = \frac{\sqrt{3}}{2}$

3. $\int \frac{x^2}{x^2+3}\,dx = \int \frac{x^2+3-3}{x^2+3}\,dx = \int \frac{x^2+3}{x^2+3}\,dx - 3\int \frac{1}{x^2+3}\,dx$ a-rule

"Slip-in Slip-out"

$= x - \frac{3}{\sqrt{3}}\arctan\left(\frac{x}{\sqrt{3}}\right) + C$

$= x - \sqrt{3}\arctan\left(\frac{x}{\sqrt{3}}\right) + C$