p-Series Test

The p-series of the form $\sum_{n=1}^{\infty} \frac{1}{n^p} = 1 + \frac{1}{2^p} + \frac{1}{3^p} + \frac{1}{4^p} + \ldots$ $\begin{cases} \text{converges if } p > 1 \\ \text{diverges if } p \leq 1 \end{cases}$

USED: For p-series exactly of the form above. Most commonly partnered together with a Comparison Test.

NOTE: Using the p-Series Test is a very quick and straightforward justification.

WARNING: Be careful to understand the difference between the Geometric Series Test and this p-Series Test. Make sense of the value or purpose of $|r|$ and p for each convergence test. How can that help you memorize each test and the tests' size arguments?

APPROACH:

- Recognize the given series in this p-Series form. Notice when the base is changing and the power is a fixed real number.
- Pick off the power p. State clearly what the value p equals.
- Determine and then state if p is greater than 1 or less or equal to 1.

EXAMPLES: Determine and state whether each of the following series **converges** or **diverges**. Name any convergence test(s) that you use, and justify all of your work.

1. $\sum_{n=1}^{\infty} \frac{1}{n^7} = 1 + \frac{1}{2^7} + \frac{1}{3^7} + \frac{1}{4^7} + \ldots$ Convergent p-Series with $p = 7 > 1$.

2. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} = \sum_{n=1}^{\infty} \frac{1}{n^{\frac{1}{2}}}$ Divergent p-Series with $p = \frac{1}{2} < 1$.

3. $\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \ldots$ Divergent Harmonic p-Series with $p = 1$.

4. $\sum_{n=1}^{\infty} \frac{1}{n^{\frac{5}{2}}}$ Convergent p-Series with $p = \frac{5}{2} > 1$.

5. $\sum_{n=1}^{\infty} \frac{1}{n^{.99}}$ Divergent p-Series with $p = .99 < 1$.