Math 121, Sections 01, 02, 03, Fall 2021

Homework #17

Due **Wednesday, November 10th** in Gradescope by 11:59 pm ET

Goal: Exploring more of the Relationship between Power Series and functions, including the Definition of the Taylor/MacLaurin Series and Substitution into known Series. Also SUMS of series which are not Geometric.

Use the *Definition* of the Taylor Series ("Chart Method") to find the **first four nonzero terms** of the series for $f(x)$ centered at the given value.

1. $f(x) = \frac{2}{3}x^{\frac{3}{2}}$ for $a = 4$

2. $f(x) = \frac{1}{1+x}$ for $a = 2$

Find the MacLaurin Series for $f(x)$ using two different methods. **First**, using the *Definition* of the MacLaurin Series ("Chart Method"). **Second**, use Substitution into a known series. Your answers should be in Sigma notation.

3. $f(x) = e^{-2x}$

4. $f(x) = \sinh x$ Hint: what is the formula for $\sinh x$?

Use Series to compute each of the following. Your answers should be in Sigma notation. State the Radius for each problem.

5. $\frac{d}{dx}\left(x^3 \arctan(7x)\right)$

6. $\int x^3 \arctan(7x)\, dx$

7. $\frac{d}{dx} x^2 \ln(1+6x)$

8. $\int x^4 e^{-x^3}\, dx$

Find the Sum of each of the following Series, which do converge.

9. $\sum_{n=0}^{\infty} \frac{7^n}{n!}$

10. $\sum_{n=0}^{\infty} \frac{(-1)^n 5^n}{n!}$

11. $\sum_{n=0}^{\infty} (-1)^n \frac{x^{4n}}{n!}$

12. $\sum_{n=0}^{\infty} \frac{(-1)^n\ \pi^{2n}}{6^{2n}\ (2n)!}$

13. $\sum_{n=0}^{\infty} \frac{3^n}{5^n\ n!}$

14. $\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1}$

15. $1 - \ln 2 + \frac{(\ln 2)^2}{2!} - \frac{(\ln 2)^3}{3!} + \ldots$

16. $3 + \frac{9}{2!} + \frac{27}{3!} + \frac{81}{4!} + \ldots$

17. Use Series to Compute $\lim_{x \to 0} \frac{1 - \cos x}{1 + x - e^x}$ Check your answer using L'Hôpital's Rule

REGULAR OFFICE HOURS

Monday: 1:00–3:00 pm

9–10:30 pm TA Mia, SMUDD 207

Tuesday: 12:00–4:00 pm

6–7:30 pm TA Ian, SMUDD 207

7:30–9:00 pm TA Karime, SMUDD 207

Wednesday: 1:00-3:00 pm

6–7:30 pm TA Ian, SMUDD 207

7:30–9:00 pm TA Daksha, SMUDD 207

Thursday: none for Professor

1–2:30 pm TA Mia, SMUDD 207

7:30–9:00 pm TA Daksha, SMUDD 207

Friday: 12:00–2:00 pm

2:30–4:00 pm TA Karime, SMUDD 014**

Challenge: Commit at least one hour per day, beyond homework, to studying Math.