

## Homework #16

Due **Friday, November 5th** in Gradescope by 11:59 pm ET

**Goal:** Exploring more of the Relationship between Power Series and functions, including differentiation and integration of Power Series.

For #1 – 4, find the Power Series Representation for the following functions and determine the Radius of Convergence  $R$ .

1.  $f(x) = \frac{1}{1+x^2}$

2.  $f(x) = \frac{x^2}{x^4+16}$

3.  $f(x) = \frac{1}{(1+x)^2}$  Hint:  $\frac{1}{(1+x)^2} = \frac{d}{dx} \left( -\frac{1}{1+x} \right)$

4.  $f(x) = \arctan(x^2)$

(a) First, Use this fact  $\arctan(x^2) = \int \frac{2x}{1+x^4} dx$ . Show that  $C = 0$ .

(b) Next, use *Substitution* into the known Series formula for  $\arctan x$ . Show (a)/(b) match.

5. Prove the Power Series Representation formula for  $\ln(1+x)$ . Yes, show that  $C = 0$ .

6. Prove the Power Series Representation formula for  $\arctan x$ . Yes, show that  $C = 0$ .

7. Find Power Series Representation for  $\ln(5-x)$ . Solve for  $C$  and the Radius  $R$ .

8. You do **not** need to state the Radius. Answers should be in Sigma notation  $\sum_{n=0}^{\infty}$  here.

You may use the fact that  $\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$  without extra justification.

(a) Use the Definition to compute the MacLaurin Series for  $F(x) = \cos x$ .

(b) Use Differentiation to compute the Series for  $F(x) = \cos x$ .

(c) Use Integration to compute the Series for  $F(x) = \cos x$ .

Hints: yes, you should solve for  $+C$ . yes,  $C$  should equal 1. Show why  $C = 1$ .

# REGULAR OFFICE HOURS

**Monday: 1:00–3:00 pm**

9–10:30 pm TA Mia, SMUDD 207

**Tuesday: 12:00–4:00 pm**

6–7:30 pm TA Ian, SMUDD 207

7:30–9:00 pm TA Karime, SMUDD 207

**Wednesday: 1:00–3:00 pm**

6–7:30 pm TA Ian, SMUDD 207

7:30–9:00 pm TA Daksha, SMUDD 207

**Thursday: none for Professor**

1–2:30 pm TA Mia, SMUDD 207

7:30–9:00 pm TA Daksha, SMUDD 207

**Friday: 12:00–2:00 pm**

2:30–4:00 pm TA Karime, SMUDD 014\*\*

Pay careful attention to details here.

Manipulating power series requires a balance  
of memory and technical skill.