Math 121, Sections 01, 02, 03, Fall 2021

Homework #13

Due Wednesday, October 20th in Gradescope by 11:59 pm ET

Goal: Exploring Convergence of Infinite Series. Focus on Alternating Series Test, and Ratio Test. We will also focus on fluency of training, using multiple tests.

1. Consider $\sum_{n=1}^{\infty} \frac{n+1}{n^2+4n+7}$. Use **two** Different methods, namely the Integral Test and the Limit Comparison Test, to prove that this series Diverges. You can skip checking the Integral Test preconditions here this time. yay!

Determine if the given Alternating series Converges or Diverges.

2.
$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{3n+1}$$
 3. $\sum_{n=1}^{\infty} (-1)^n \frac{3n-1}{2n+1}$

Determine if the given series is Absolutely Convergent or Divergent.

4.
$$\sum_{n=1}^{\infty} \frac{n}{5^n}$$
 5. $\sum_{n=1}^{\infty} \frac{(-3)^n}{(2n+1)!}$ 6. $\sum_{n=1}^{\infty} \frac{n!}{100^n}$

7.
$$\sum_{n=1}^{\infty} \frac{n!}{n^n}$$
 8. $\sum_{n=1}^{\infty} \frac{n^{100} \ 100^n}{n!}$ 9. $\sum_{n=1}^{\infty} \frac{(2n)!}{(n!)^2}$

10. Consider the series $\sum_{n=1}^{\infty} \frac{\ln n}{n}$.

(a) Show that n^{th} Term Divergence Test is **Inconclusive**.

(b) Show that the Ratio Test is **Inconclusive**.

(c) Show that the series Diverges using the Integral Test. Skip checking the 3 preconditions here. **Note:** This is an example where the terms approach 0 but the series Diverges.

11. Prove that
$$\sum_{n=1}^{\infty} \frac{6}{n^6}$$
 is Convergent by using the Limit Comparison Test

Note that this work will be a sample proof of the fact that *Constant multiple of a Convergent* series is *Convergent*.

12. Show that
$$\sum_{n=1}^{\infty} \frac{e^n}{n^2}$$
 Diverges using **two** Different methods.

REGULAR OFFICE HOURS

Monday: 1:00–3:00 pm

9–10:30 pm TA Mia, SMUDD 207

Tuesday: 12:00–4:00 pm

6–7:30 pm TA Ian, SMUDD 207

 $7{:}30{-}9{:}00~\mathrm{pm}$ TA Karime, SMUDD 207

Wednesday: 1:00-3:00 pm

6-7:30 pm TA Ian, SMUDD 207

 $7{:}30{-}9{:}00~\mathrm{pm}$ TA Daksha, SMUDD 207

Thursday: none for Professor

1–2:30 pm TA Mia, SMUDD 207

7:30–9:00 pm TA Daksha, SMUDD 207

Friday: 12:00-2:00 pm

2:30-4:00 pm TA Karime, SMUDD 014**

Train your Convergence Tests Daily