Math 121 Midterm Exam #3 November 12-15, 2020

Due Sunday, November 15, in Gradescope by 11:59 pm EDT

• This is an *Open Notes* Exam. You can use materials, homeworks problems, lecture notes, etc. that you manually worked on.

• There is **NO** *Open Internet* allowed. You can only access our Main Course Webpage.

• You are not allowed to work on or discuss these problems with anyone. You can ask a few small, clarifying, questions about instructions in Office Hours, but these problems will not be solved.

• Submit your final work in Gradescope in the Exam 3 entry.

• Please *show* all of your work and *justify* all of your answers.

1. [24 Points] Find the **Interval** and **Radius** of Convergence for each of the following power series. Analyze carefully and with full justification.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n\ (3x+4)^n}{(n+7)^2 \cdot 8^n}$ (b) $\displaystyle\sum_{n=1}^{\infty} (2n)!\ (\ln n)\ (x-7)^n$ (c) $\displaystyle\sum_{n=1}^{\infty} \frac{x^{3n-1}}{n^n}$

2. [18 Points] Your answers should all be in sigma notation $\displaystyle\sum_{n=0}^{\infty}$ here.

(a) Write the MacLaurin Series for $f(x) = x^3 \arctan(7x)$ and **State** the Radius of Convergence.

(b) Use your Series in part (a) to compute $\dfrac{d}{dx}\left(x^3 \arctan(7x)\right)$.

(c) Use your Series in part (a) to compute $\displaystyle\int x^3 \arctan(7x)\ dx$.

3. [10 Points] Use Series to **Estimate** $\displaystyle\int_0^1 x^2 e^{-x^3}\ dx$ with error less than $\dfrac{1}{50}$. Justify all details.

4. [20 Points] Find the **sum** for each of the following series (which do converge). Simplify.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n \ \pi^{2n}}{9^n \ (2n+1)!}$ (b) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^{n+1} \ 2^{n+1}(\ln 9)^n}{n!}$ (c) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^{n+1} \ \pi^{2n+1}}{9 \ (2n)!}$

(d) $-1 + \frac{1}{3} - \frac{1}{5} + \frac{1}{7} - \frac{1}{9} + \ldots$ (e) $\displaystyle\sum_{n=0}^{\infty} \frac{1}{3! \ \pi^n}$ (f) $\frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \frac{1}{7} - \ldots$

5. [16 Points] Do not just write a formula. You do **not** need to state the Radius. Your answers should all be in Sigma notation $\displaystyle\sum_{n=0}^{\infty}$ here.

You may use the fact that $\boxed{\sin x = \displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}}$ without extra justification.

(a) Demonstrate one method to compute the MacLaurin Series for $F(x) = \cos x$.

(b) Demonstrate a second, **different**, method to compute the MacLaurin Series for $F(x) = \cos x$.

(c) Demonstrate a third, **different**, method to compute the MacLaurin Series for $F(x) = \cos x$.
Hint: yes, you should solve for $+C$.
Hint: yes, C should equal 1. Show why $C = 1$.

6. [12 Points] Consider the Parametric Curve given by $x = e^t + \dfrac{1}{1+e^t}$ and $y = 2\ln\left(1+e^t\right)$.

Hint: $\dfrac{dx}{dt} = e^t - \dfrac{e^t}{(1+e^t)^2}$ and $\dfrac{dy}{dt} = \dfrac{2e^t}{(1+e^t)}$

(a) Show that the Slope $\dfrac{dy}{dx}$ for this parametric curve when $t = 0$ is equal to $\boxed{\dfrac{4}{3}}$

(b) Show that the Arclength of this parametric curve for $0 \le t \le \ln 3$ is equal to $\boxed{\dfrac{9}{4}}$