Review Packet for Exam #2

Math 121-D. Benedetto

Integrals: Compute each of the following integrals, or else show that it diverges.

1. $\displaystyle\int_3^\infty \frac{1}{x^2-4x+7}\,dx$
2. $\displaystyle\int_e^\infty \frac{1}{x(\ln x)^3}\,dx$
3. $\displaystyle\int_0^\infty \frac{1}{(x+2)(2x+5)}\,dx$
4. $\displaystyle\int_7^\infty \frac{1}{x^2-8x+19}\,dx$
5. $\displaystyle\int_2^\infty \frac{1}{x^2-2x+4}\,dx$
6. $\displaystyle\int \frac{1}{x^2+2x+2}\,dx$
7. $\displaystyle\int_0^{\frac{\pi}{2}} \tan x\,dx$
8. $\displaystyle\int_3^4 \frac{1}{(x-4)^2}\,dx$
9. $\displaystyle\int_1^2 \frac{1}{x\ln x}\,dx$
10. $\displaystyle\int_0^1 \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$
11. $\displaystyle\int_1^\infty \frac{\ln x}{x}\,dx$
12. $\displaystyle\int \frac{2x-5}{x^2+2x+2}\,dx$
13. $\displaystyle\int_0^1 \frac{e^x}{\sqrt{e^x-1}}\,dx$
14. $\displaystyle\int_0^1 \ln x\,dx$
15. $\displaystyle\int_0^1 \frac{1}{(1-x^2)^{\frac{3}{2}}}\,dx$
16. $\displaystyle\int_{-\infty}^\infty \frac{1}{x^2-6x+10}\,dx$

17. $\displaystyle\int_{-5}^{0} \frac{x}{x^2+4x-5}\,dx$

18. $\displaystyle\int \frac{x^5+2}{x^2-1}\,dx$

19. $\displaystyle\int_{0}^{6} \frac{1}{(x-2)^2}\,dx$

20. $\displaystyle\int_{0}^{\infty} \frac{1}{x^2+3x+2}\,dx$

21. $\displaystyle\int_{0}^{\frac{\pi}{2}} \tan^2 x\,dx$

22. $\displaystyle\int_{0}^{2} \frac{1}{(4-x^2)^{\frac{3}{2}}}\,dx$

23. $\displaystyle\int \frac{4x^2+7x+6}{(x+2)(x^2+4)}\,dx$

24. $\displaystyle\int_{1}^{\infty} \frac{1}{x(x+1)}\,dx$

25. $\displaystyle\int_{-3}^{-2} \frac{1}{x^2-4}\,dx$

26. $\displaystyle\int_{0}^{1} \arcsin x\,dx$ (leads to improper integral)

27. $\displaystyle\int \frac{x^3+7x+1}{x^2+1}\,dx$

28. $\displaystyle\int \frac{x^4+x^3+2x^2+6x+2}{(x+1)(x^2+1)}\,dx$

29. $\displaystyle\int_{0}^{1} \frac{e^{\frac{1}{x}}}{x^2}\,dx$

30. $\displaystyle\int_{-1}^{0} \frac{e^{\frac{1}{x}}}{x^2}\,dx$

31. $\displaystyle\int_{-\infty}^{\infty} \frac{x^2}{9+x^6}\,dx$

32. $\displaystyle\int_{2}^{\infty} \frac{x}{e^{3x}}\,dx$

33. $\displaystyle\int_{0}^{e} \frac{\ln x}{\sqrt{x}}\,dx$

Sequences: For each of the following sequences, decide whether it converges or diverges. If it converges, compute its limit.

34. $\left\{\dfrac{1+n-7n^4}{3n^4+8n^3+9}\right\}_{n=1}^{\infty}$

35. $\left\{\dfrac{n^3}{(n+1)^3}\right\}_{n=1}^{\infty}$

36. $\left\{\left(\dfrac{n-5}{n}\right)^n\right\}_{n=1}^{\infty}$

37. $\left\{\dfrac{(2n+3)!}{(2n+5)!}\right\}_{n=1}^{\infty}$

38. $\left\{\arctan(n^2+1)\right\}_{n=1}^{\infty}$

39. $\left\{\dfrac{\sqrt{n}}{(\ln n)^2}\right\}_{n=1}^{\infty}$

40. $\left\{n^{\frac{1}{n}}\right\}_{n=1}^{\infty}$

41. $\left\{n\sin\left(\dfrac{1}{n}\right)\right\}_{n=1}^{\infty}$

42. $\left\{\left(\dfrac{n+1}{n}\right)^n\right\}_{n=1}^{\infty}$

43. $\left\{\left(\dfrac{n}{n+1}\right)^n\right\}_{n=1}^{\infty}$

Series: Find the **sum** for each of the following series (all of which converge):

44. $\displaystyle\sum_{n=1}^{\infty}\frac{2^n+3^n}{6^n}$

45. $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^{n+1}2^{n-1}}{3^{n+1}}$

46. $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n 3^{n+2}}{2^{4n-1}}$

47. $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n 4^n}{3^{2n-1}}$

More Series: Determine whether each of the following series **converge** or **diverge**. Name any convergence test(s) you use, and justify all of your work

48. $\displaystyle\sum_{n=1}^{\infty} \frac{2n^3 - \ln n}{5n^3 + 9}$

49. $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n^2}$

50. $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n} + 3}{4n^2 - 2}$

51. $\displaystyle\sum_{n=1}^{\infty} \frac{n^{19} + 40n^6 + 4n^3 + 19}{4 + 17n^5 + n^{20}}$

52. $\displaystyle\sum_{n=2}^{\infty} \frac{e^n}{\ln n}$

53. $\displaystyle\sum_{n=1}^{\infty} \frac{5}{n^5} + \frac{1}{5^n}$

54. $\displaystyle\sum_{n=1}^{\infty} \frac{1 + 3n^3}{n^5}$

55. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^7}$

56. $\displaystyle\sum_{n=1}^{\infty} \frac{\arctan n}{1 + n^2}$

57. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2 + 1}{\arctan n}$

58. $\displaystyle\sum_{n=1}^{\infty} \frac{2n + 5}{5n^3 + 3n^2}$

59. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2 + 5n - 3}$

60. $\displaystyle\sum_{n=1}^{\infty} \frac{\pi}{\arctan(2n)}$

61. $\displaystyle\sum_{n=1}^{\infty} 3 + \frac{1}{3^n}$

62. $\displaystyle\sum_{n=1}^{\infty} e^{\frac{1}{n}}$

63. $\displaystyle\sum_{n=1}^{\infty} \frac{6}{n^6} + \frac{1}{(n+1)^6}$

64. $\displaystyle\sum_{n=1}^{\infty} \cos^2\left(\frac{\pi n^2 + n}{n^2 + 7}\right)$

65. $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{\cos^2(\pi n^2 + 1)}{n^2 + 7}$

Even More Series: Determine whether each of the following series **converges absolutely**, **converges conditionally**, or **diverges**. Name any convergence test(s) you use, and justify all of your work.

66. $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{5n + 2}$

67. $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n^3 + 6n}{n^8 + 1}$

68. $\displaystyle\sum_{n=1}^{\infty} \frac{5^{2n}}{(2n+1)!\ \ln n}$

69. $\displaystyle\sum_{n=1}^{\infty} \frac{n!\ n^6\ n^n}{10^{4n} e^{2n}}$

70. $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{1}{4n + 3}$

71. $\displaystyle\sum_{n=1}^{\infty} \frac{(n!)^3 e^{2n}}{(3n)!\ n^n}$

72. $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \frac{\arctan n}{n^7 + n}$

73. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n (3n)!\ n^2}{8^n\ (n!)^2\ n^n}$

74. $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n^3 + 7}{n^7 + 3}$

75. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n\ (\ln n)\ \pi^n\ (2n)!}{n^n\ 4^n\ n!}$