

- This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or other aids are permitted.
- You need *not* simplify algebraically complicated answers. However, numerical answers such as $\sin\left(\frac{\pi}{6}\right)$, $4^{\frac{3}{2}}$, $\sinh(\ln 3)$, $e^{\ln 4}$, $\ln(e^7)$, or $e^{3\ln 3}$ should be simplified.
- Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

1. [8 Points]

Use implicit differentiation to **PROVE** that $\frac{d}{dx} \arcsin(5x) = \frac{5}{\sqrt{1-25x^2}}$.

2. [32 Points] Evaluate each of the following **limits**. Please justify your answers. Be clear if the limit equals a value, $+\infty$ or $-\infty$, or Does Not Exist.

(a) $\lim_{x \rightarrow 0} \frac{3xe^x - \arctan(3x)}{x + \ln(1-x)}$

(b) $\lim_{x \rightarrow 0} (1 + \ln(1-3x))^{\frac{1}{x}}$

(c) $\lim_{x \rightarrow \infty} \left[1 + \arcsin\left(\frac{1}{x}\right) + \sin\left(\frac{1}{x}\right) \right]^x$

3. [40 Points] Compute each of the following integrals. Please simplify your answer.

(a) $\int_0^1 x \arctan x \, dx$

(b) $\int_1^{\sqrt{3}} \frac{x^2}{\sqrt{4-x^2}} \, dx$

(c) $\int x^3 \sqrt{1-x^2} \, dx$ using a **trigonometric substitution**.

(d) $\int_1^{e^3} (\ln x)^2 \, dx$

4. [20 Points] Compute each of the following **indefinite integrals**.

(a) $\int \frac{1}{(1+x^2)[5+(\arctan x)^2]} dx$

(b) $\int \frac{1}{(1+x^2)^{\frac{5}{2}}} dx$

OPTIONAL BONUS

Do not attempt these unless you are completely done with the rest of the exam.

OPTIONAL BONUS #1 Compute the following **indefinite integral**.

1. $\int \sec^3 x dx$

OPTIONAL BONUS #2 Compute the following **indefinite integral**.

2. $\int \frac{1}{1+3\sin^2 x} dx$

OPTIONAL BONUS #3 Show that $\cos(\arctan(\sin(\cot^{-1} x))) = \sqrt{\frac{x^2+1}{x^2+2}}$