

Math 121 Midterm Exam #1 September 17-20, 2020
Due Sunday, September 20, in Gradescope by 11:59 pm EDT

- This is an *Open Notes* Exam. You can use materials, homeworks problems, lecture notes, etc. that you manually worked on.
- There is **NO** *Open Internet* allowed. You can only access our Main Course Webpage.
- You are not allowed to work on or discuss these problems with other people. You can ask a few small, clarifying, questions in Office Hours, but these problems will not be solved.
- Submit your final work in Gradescope in the Exam 1 entry.
- Please *show* all of your work and *justify* all of your answers.

1. [8 Points] Use Trig. Substitution to **prove** that $\int \frac{1}{4+x^2} dx = \frac{1}{2} \arctan\left(\frac{x}{2}\right) + C$

2. [22 Points] Evaluate the following **limits**. Please justify your answer. Be clear if the limit equals a value, $+\infty$ or $-\infty$, or Does Not Exist. Simplify.

(a) $\lim_{x \rightarrow 0} \frac{\cosh(3x) - \arctan(2x) + 2x - 1}{\arcsin x - e^x + 1}$

(b) $\lim_{x \rightarrow \infty} \left(1 - \frac{2}{x^3}\right)^{x^3}$

3. [30 Points] Compute each of the following definite integrals. Please simplify.

(a) Show that $\int_1^{e^2} \sqrt{x} \cdot \ln x \, dx = \frac{8e^3 + 4}{9}$

(b) Show that $\int_3^9 \frac{1}{\sqrt{x}(x+9)} \, dx = \frac{\pi}{18}$

(c) Show that $\int_0^{\ln \sqrt{3}} \frac{e^x}{\sqrt{4 - e^{2x}}} \, dx = \frac{\pi}{6}$

4. [40 Points] Compute each of the following indefinite integrals. Please simplify.

(a) $\int \frac{x^2 + x + 1}{x^2 + 3} \, dx$

(b) $\int (x + 1) \arcsin x \, dx$

(c) $\int \frac{1}{(9 + x^2)^{\frac{7}{2}}} \, dx$ Hint: $3^6 = 729$