Math 121 Exam 3
April 24, 2026

- This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or other aids are permitted.

- Numerical answers such as $\sin\left(\frac{\pi}{6}\right)$, $4^{\frac{3}{2}}$, $e^{\ln 4}$, $\ln(e^7)$, $e^{3\ln 3}$, or $\arctan\sqrt{3}$ should be simplified.

- Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

1. [20 Points] Interval and Radius of Convergence. Analyze carefully, with full justification.

(a) Find the **Interval** and **Radius** of Convergence for $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n\,(3x+4)^n}{n^3 \cdot 8^n}$

(b) Show that the MacLaurin Series for $\cos x$ has an *Infinite* Radius of Convergence.

2. [20 Points] Find the MacLaurin Series for each of the functions. Also **STATE** the Radius of Convergence for each series. Answers should be in sigma notation $\displaystyle\sum_{n=0}^{\infty}$. Simplify.

(a) $\dfrac{d}{dx}\left(8x^4\sin(8x)\right)$ (b) $\displaystyle\int \ln\left(1+9x^2\right)\,dx$

3. [12 Points] Use Series to Estimate $\displaystyle\int_0^1 x^3 e^{-x^2}\,dx$ with error less than $\dfrac{1}{50}$. Justify. Simplify. Tip: a common denominator for 4 and 6 and 16 is 48.

4. [26 Points] Find the **sum** for each of the following convergent series. Simplify, if possible.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n\ \pi^{2n}}{3^{2n}\ (2n+1)!}$ (b) $-1+\frac{1}{2}-\frac{1}{3}+\frac{1}{4}-\frac{1}{5}+\frac{1}{6}-\ldots$ (c) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n\ \pi^{2n+1}}{4^n\ (2n)!}$

(d) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^{n+1}\ (\ln 9)^n}{2^n \cdot n!}$ (e) $4+4-\frac{4}{3}+\frac{4}{5}-\frac{4}{7}+\frac{4}{9}-\ldots$ (f) $-\frac{\pi^2}{2!}+\frac{\pi^4}{4!}-\frac{\pi^6}{6!}+\frac{\pi^8}{8!}-\ldots$

5. [10 Points] Use Series to compute the following Limit $\displaystyle\lim_{x\to 0} \frac{1+4x-e^{4x}}{1-\cos(3x)}$

6. [12 Points] Find the MacLaurin Series Representation for $\ln\left(8+x^3\right)$.
You **MUST** use Integration of Series, and the following Hint formula.

Hint: $\boxed{\ln\left(8+x^3\right) = \int \frac{3x^2}{\cancel{8}+x^3}\,dx \xrightarrow{\text{P.S.}} = \int 3x^2\left(\frac{1}{8+x^3}\right)\,dx = \ldots}$

Yes, solve for C and STATE the Radius of Convergence.

OPTIONAL: Feel free to check your answer using a substitution.

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OPTIONAL BONUS

Do not attempt these unless you are completely done with the rest of the exam.

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OPTIONAL BONUS #1 Create a Power Series with Interval of Convergence $I = \left(-\frac{4}{3}, \frac{1}{5}\right]$ Continue on to justify that your series satisifies this challenge.