Math 121   Midterm Exam #3   April 26-28, 2020

- You may **only** access your class notes. **No** books, calculators, cell phones, people, or webpages.
- Numerical answers such as $\sin\left(\frac{\pi}{6}\right)$, $\ln(e^7)$, $e^{3\ln 3}$, $\arctan\sqrt{3}$ or $\cosh(\ln 3)$ should be simplified.
- Please *show* all of your work and *justify* all of your answers.
- You may work for no more than 24 consecutive hours.
- When done, immediately upload to the EXAM 3 entry in Gradescope. **TAG** problems.

1. [22 Points] Find the **Interval** and **Radius** of Convergence for the following power series. Analyze carefully and with full justification.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n\,(5x+1)^n}{(n+1)\,9^n}$ (b) $\displaystyle\sum_{n=1}^{\infty} n^n(x-3)^n$ (c) $\displaystyle\sum_{n=1}^{\infty} \frac{x^{2n+1}}{n!}$

2. [20 Points] Your answers should all be in sigma notation $\displaystyle\sum_{n=0}^{\infty}$ here.

(a) Write the MacLaurin Series for $f(x) = \dfrac{x^4}{1+7x}$. **State** the Radius of Convergence.

(b) Write the MacLaurin Series for $f(x) = x^3\sin\left(x^2\right)$. **State** the Radius of Convergence.

(c) Use your Series in part (b) to compute $\displaystyle\int x^3\sin\left(x^2\right)\,dx$.

(d) Write the MacLaurin Series for $f(x) = x^2\ln(1+5x)$. **State** the Radius of Convergence.

(e) Use your Series in part (d) to compute $\dfrac{d}{dx}\left[x^2\ln(1+5x)\right]$.

3. [10 Points] Justify all details.

(a) **Estimate** $\dfrac{1}{\sqrt{e}}$ with error less than $\dfrac{1}{200}$.

(b) **Estimate** $\sin(1)$ with error less than $\dfrac{1}{1000}$. Tip: $7! = 5040$.

4. [22 Points] Find the **sum** for each of the following series (which do converge). Simplify.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n\ \pi^{2n}}{6^{2n}\ (2n+1)!}$ (b) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n\ (\ln 9)^n}{2^n\ n!}$ (c) $2 - \frac{2}{2} + \frac{2}{3} - \frac{2}{4} + \frac{2}{5} - \ldots$

(d) $-\frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \ldots$ (e) $\displaystyle\sum_{n=0}^{\infty} \frac{1}{e^n}$ (f) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^{n+1}\ \pi^{2n}}{2^{4n}\ (2n)!}$

5. [16 Points] Do not just write a formula. You do **not** need to state the Radius. Your answers should all be in Sigma notation $\displaystyle\sum_{n=0}^{\infty}$ here.

(a) Demonstrate one method to compute the MacLaurin Series for $F(x) = \ln(1+x)$.

(b) Demonstrate a second, **different**, method to compute the MacLaurin Series for $F(x) = \ln(1+x)$.

(c) Demonstrate one method to compute the MacLaurin Series for $G(x) = \sinh x$.

(d) Demonstrate a second, **different**, method to compute the MacLaurin Series for $G(x) = \sinh x$.

6. [10 Points] Consider the Parametric Curve given by $x = \frac{e^{2t}}{2} - \frac{t^3}{3}$ and $y = 2te^t - 2e^t$.

(a) Compute the derivative $\frac{dy}{dx}$ for the curve when $t = 1$.

(b) Compute the Arclength of this parametric curve for $0 \le t \le 1$.

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OPTIONAL BONUS

Do not attempt this unless you are completely done with the rest of the exam.

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OPTIONAL BONUS #1 Demonstrate a third, **different**, method than in Problem 5 above, to compute the MacLaurin Series for $F(x) = \ln(1+x)$.