**Math 121 Midterm Exam #3 April 25, 2014**

• This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or other aids are permitted.

• You need *not* simplify algebraically complicated answers. However, numerical answers such as $\sin\left(\frac{\pi}{6}\right)$, $4^{\frac{3}{2}}$, $e^{\ln 4}$, $\ln(e^7)$, $e^{3\ln 3}$, or $\cosh(\ln 3)$ should be simplified.

• Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

**1.** [15 Points] Find the **Interval** and **Radius** of Convergence for the following power series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n (4x-3)^n}{n^2\ 7^n}$. Analyze carefully and with full justification.

**2.** [15 Points] Find the **MacLaurin series** representation for each of the following functions. **State** the Radius of Convergence for each series. Your answer should be in sigma notation $\displaystyle\sum_{n=0}^{\infty}$ .

(a) $f(x) = \dfrac{x^2}{1+6x}$ (b) $f(x) = x^7 \sin(x^2)$ (c) $f(x) = x \arctan(3x)$

**3.** [15 Points]

(a) Write the MacLaurin Series representation for $f(x) = x^4 e^{-x^3}$.

(b) Use the MacLaurin Series representation for $f(x) = x^4 e^{-x^3}$ from part (a) to

$$\textbf{Estimate} \quad \int_0^1 x^4 e^{-x^3}\, dx \quad \text{with error less than } \frac{1}{10}.$$

Justify in words that your error is indeed less than $\dfrac{1}{10}$.

**4.** [15 Points] Find the **sum** for each of the following series.

(a) $1 - \dfrac{1}{2} + \dfrac{1}{3} - \dfrac{1}{4} + \dfrac{1}{5} - \dfrac{1}{6} + \ldots = \displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{n+1}$ (b) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n\ \pi^{2n}}{9^n\ (2n)!}$

(c) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n\ (\ln 9)^n}{2^n\ n!} = 1 - \frac{\ln 9}{2} + \frac{(\ln 9)^2}{2^2 \cdot 2!} - \frac{(\ln 9)^3}{2^3 \cdot 3!} + \frac{(\ln 9)^4}{2^4 \cdot 4!} + \ldots$ (d) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n\ \pi^{2n}}{3^{2n}\ (2n+1)!}$

(e) $$\sum_{n=0}^{\infty} \frac{(-1)^n \left(\frac{1}{\sqrt{3}}\right)^{2n+1}}{2n+1} = \sum_{n=0}^{\infty} \frac{(-1)^n}{(\sqrt{3})^{2n+1}(2n+1)}$$
$$= \frac{1}{\sqrt{3}} - \frac{1}{(\sqrt{3})^3\ 3} + \frac{1}{(\sqrt{3})^5\ 5} - \frac{1}{(\sqrt{3})^7\ 7} + \frac{1}{(\sqrt{3})^9\ 9} - \ldots$$

**5.** [20 Points] Volumes of Revolution

(a) Consider the region bounded by $y = \arctan x$, $y = 4$, $x = 0$, and $x = 1$. Rotate this region about the horizontal line $\boxed{y = -1}$. Set-up, **BUT DO NOT EVALUATE!!**, the integral to compute the volume of the resulting solid using the Washer Method. Sketch the solid, along with one of the approximating washers.

(b) Consider the region bounded by $y = e^x + 1$, $y = \ln x$, $x = 1$ and $x = e$. Rotate this region about the vertical line $\boxed{x = -2}$. Set-up, **BUT DO NOT EVALUATE!!**, the integral to compute the volume of the resulting solid using the Cylindrical Shells Method. Sketch the solid, along with one of the approximating shells.

(c) Consider the region bounded by $y = x^2 + 3$, $y = x$, $x = 0$ and $x = 2$. Rotate this region about the vertical line $\boxed{x = 7}$. Set-up, **BUT DO NOT EVALUATE!!**, the integral to compute the volume of the resulting solid using the Cylindrical Shells Method. Sketch the solid, along with one of the approximating shells.

(d) Consider the region bounded by $y = \sin x$, $y = 0$, $x = 0$ and $x = \pi$. Rotate this region about the $\boxed{y\text{-axis}}$. **<u>COMPUTE</u>** the volume of the resulting solid using the Cylindrical Shells Method. Sketch the solid, along with one of the approximating shells.

**6.** [20 Points]

(a) Consider the Parametric Curve represented by $x = t + \dfrac{1}{t}$ and $y = \ln(t^2)$.

**<u>COMPUTE</u>** the **arclength** of this parametric curve for $1 \le t \le 4$.

(b) Consider a different parametric curve represented by $x = \sin t - t\cos t$ and $y = \cos t + t\sin t$.

Set-up, **BUT DO NOT EVALUATE!!**, the definite integral representing the **surface area** obtained by rotating this curve about the $x$-**axis**, for $\dfrac{\pi}{3} \le t \le \pi$.

*****************************************************************************

# OPTIONAL BONUS

Do not attempt this unless you are completely done with the rest of the exam.

*****************************************************************************

**OPTIONAL BONUS #1** Find the MacLaurin Series for $f(x) = xe^{-x^2}$. Use this series to determine the $8^{th}$ and $9^{th}$ derivatives for $f(x) = xe^{-x^2}$ at $x = 0$.

Do not compute out these derivatives manually.