Math 12 Midterm Exam #3 Compact Version April 27, 2011

• This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or other aids are permitted.

• You need *not* simplify algebraically complicated answers. However, numerical answers such as $\sin\left(\frac{\pi}{6}\right)$, $4^{\frac{3}{2}}$, $e^{\ln 4}$, $\ln(e^7)$, or $e^{3\ln 3}$ should be simplified.

• Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

1. [15 Points] Find the **Interval** and **Radius** of Convergence for each of the following power series. Analyze carefully and with full justification.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{(2n)!\, x^{2n+1}}{(3n)!}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(3x+5)^n}{n^2 7^n}$

2. [8 Points] Consider the function $f(x)$ that satisfies the following

$$f(4) = 2 \qquad f'(4) = -3 \qquad f''(4) = \frac{6}{7} \qquad f'''(4) = -1$$

Find the **Taylor polynomial of degree 3** for $\mathbf{f(x)}$ centered at $\mathbf{a = 4}$.

3. [10 Points] Find the **MacLaurin series** representation for each of the following functions. State the Radius of Convergence for each series. Your answer should be in sigma notation $\displaystyle\sum_{n=0}^{\infty}$.

(a) $f(x) = x^2 e^{-3x}$

(b) $f(x) = x \arctan(3x)$

4. [12 Points] Use a Power Series representation for $x\ln(1+x^3)$ to estimate the given integral within the given error. Justify in words that your error is indeed less than $\frac{1}{10}$.

$$\text{Estimate} \quad \int_0^1 x\ln(1+x^3)\, dx \quad \text{with error less than } \frac{1}{10}$$

5. [15 Points] Find the **sum** for each of the following series.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n 7^n}{9^n n!}$ (b) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n}}{6^{2n}(2n)!}$ (c) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n}}{9^n (2n+1)!}$

6. [20 Points] Volumes of Revolution

(a) Consider the region bounded by $y = \cos x$, $y = e^x + 2$, $x = 0$, and $x = \dfrac{\pi}{2}$. Rotate this region about the horizontal line $\boxed{y = -3}$. Set-up, **BUT DO NOT EVALUATE!!**, the integral to compute the volume of the resulting solid using the Washer Method. Sketch the solid, along with one of the approximating washers.

(b) Consider the region bounded by $y = e^x$, $y = \ln x$, $x = 1$ and $x = 2$. Rotate this region about the $\boxed{y\text{-axis}}$. **COMPUTE** the volume of the resulting solid using the Cylindrical Shells Method. Sketch the solid, along with one of the approximating shells.

7. [20 Points] Consider the Parametric Curve represented by $x = 3 - 2t$ and $y = e^t + e^{-t}$.

(a) **Compute** the **arclength** of this parametric curve for $0 \le t \le 1$.

(b) **Compute** the **surface area** obtained by rotating this curve about the x**-axis**, for $0 \le t \le 1$.

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OPTIONAL BONUS

Do not attempt these unless you are completely done with the rest of the exam.

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OPTIONAL BONUS #1 Compute the **sum** $\displaystyle\sum_{n=0}^{\infty} \frac{n}{7^n}$

OPTIONAL BONUS #2 Compute the **sum** $\displaystyle\sum_{n=0}^{\infty} \frac{n^2}{7^n n!}$

OPTIONAL BONUS #3 Compute the sum $\displaystyle\sum_{n=0}^{\infty} \frac{n^3}{7^n}$