



Math 121 Exam 3 December 1, 2023

- This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or other aids are permitted.
- Numerical answers such as $\sin\left(\frac{\pi}{6}\right)$, $4^{\frac{3}{2}}$, $e^{\ln 4}$, $\ln(e^7)$, $e^{3\ln 3}$, or $\arctan \sqrt{3}$ should be simplified.
- Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

1. [16 Points] Find the **Interval** and **Radius** of Convergence for $\sum_{n=1}^{\infty} \frac{(-1)^n (3x+5)^n}{(3n+5)^2 \cdot 4^n}$

Analyze carefully and with full justification.

2. [22 Points] Find the MacLaurin Series for each of the functions. Also **STATE** the Radius of Convergence for each series. Answers should be in sigma notation $\sum_{n=0}^{\infty}$. Simplify.

(a) $\ln(1+9x^2)$ (b) $x^3 e^{-4x}$ (c) $\frac{d}{dx}(8x^4 \sin(8x))$ (d) $\int \frac{x^2}{8+x^3} dx$

3. [12 Points] Use MacLaurin Series to Estimate $\cos\left(\frac{1}{2}\right)$ with error less than $\frac{1}{10000}$.

Simplify.

Tips: $(16) \cdot (24) = 384$ and $6! = 720$ and $(720) \cdot (64) = 46,080$ and $(48) \cdot 8 = 384$

4. [26 Points] Find the **sum** for each of the following convergent series. Simplify, if possible.

(a) $\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n}}{3^{2n} (2n+1)!}$ (b) $-1 + \frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5} + \frac{1}{6} - \dots$ (c) $\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n+1}}{4^n (2n)!}$

(d) $\sum_{n=0}^{\infty} \frac{(-1)^{n+1} (\ln 9)^n}{2^n \cdot n!}$ (e) $4+4-\frac{4}{3}+\frac{4}{5}-\frac{4}{7}+\frac{4}{9}-\dots$ (f) $-\frac{\pi^2}{2!}+\frac{\pi^4}{4!}-\frac{\pi^6}{6!}+\frac{\pi^8}{8!}-\dots$

5. [12 Points] Use Series to compute the following Limit $\lim_{x \rightarrow 0} \frac{1 - \cos(2x)}{e^{-x} - 1 + x}$

6. [12 Points] Find the MacLaurin Series Representation for $\arctan(x^4)$.

You **MUST** use Integration of Series, and the following Hint formula.

Hint: $\arctan(x^4) = \int \frac{4x^3}{1+x^8} dx = \int 4x^3 \left(\frac{1}{1+x^8} \right) dx$ Yes, solve for C

P.S.

OPTIONAL: Feel free to check your answer using a substitution.