Math 121 Midterm Exam #3 December 6, 2019

• This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or other aids are permitted.

• Numerical answers such as $\sin\left(\frac{\pi}{6}\right)$, $4^{\frac{3}{2}}$, $e^{\ln 4}$, $\ln(e^7)$, $e^{3\ln 3}$, $\arctan\sqrt{3}$ or $\cosh(\ln 3)$ should be simplified.

• Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

1. [18 Points] Find the **Interval** and **Radius** of Convergence for the following power series. Analyze carefully and with full justification.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n\ (3x+4)^n}{n^2 \cdot 8^n}$ (b) $\displaystyle\sum_{n=1}^{\infty} n^n (x-3)^n$

2. [14 Points]

(a) Write the MacLaurin Series for $f(x) = \arctan(2x)$. **State** the Radius of Convergence. Your answer should be in sigma notation $\displaystyle\sum_{n=0}^{\infty}$.

(b) Use **Series** to show that $\displaystyle\lim_{x\to 0} \frac{\arctan(2x) - 2x}{x - (x \cdot \cos x)} = \boxed{-\frac{16}{3}}$.

3. [12 Points] Use the MacLaurin Series representation for $f(x) = x^4 e^{-x^3}$ to

$$\textbf{Estimate} \quad \int_0^1 x^4 e^{-x^3}\, dx \quad \text{with error less than } \frac{1}{10}.$$

Justify in words that your error is indeed less than $\frac{1}{10}$.

4. [24 Points] Find the **sum** for each of the following series. Simplify, if possible.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n\ \pi^{2n}}{(2n)!}$ (b) $-\frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \dots$ (c) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^{n+1}\ \pi^{2n}}{36^n\ (2n+1)!}$

(d) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n\ (\ln 8)^n}{3^{n+1}\ n!}$ (e) $-2 + \frac{2}{2} - \frac{2}{3} + \frac{2}{4} - \frac{2}{5} + \dots$ (f) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n\ \pi^{2n+1}}{(\sqrt{2}\,)^{4n}\ (2n)!}$

5. [20 Points] Justify all steps. Simplify.

(a) **Compute** the MacLaurin Series for the Hyperbolic Cosine $F(x) = \cosh x$ in any way that you wish. Your answer should be in Sigma notation $\sum_{n=0}^{\infty}$.

(b) Use your answer from part (a) to write the MacLaurin Series for $G(x) = \cosh\left(2x^3\right)$. Your answer should be in Sigma notation $\sum_{n=0}^{\infty}$.

(c) Use the Series in part (b) to compute the **twelfth** and **thirteenth** derivatives of G evaluated at $x = 0$. That is, compute $G^{(12)}(0)$ and $G^{(13)}(0)$. Do **not** simplify your answers.

6. [12 Points] Consider the Parametric Curve given by $x = \dfrac{e^{2t}}{2} - \dfrac{t^3}{3}$ and $y = 2te^t - 2e^t$.

COMPUTE the **Arclength** of this parametric curve for $0 \le t \le 1$.