Math 121 Midterm Exam #3 December 5, 2018

• This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or other aids are permitted.

• Numerical answers such as $\sin\left(\frac{\pi}{6}\right)$, $4^{\frac{3}{2}}$, $e^{\ln 4}$, $\ln(e^7)$, $e^{3\ln 3}$, $\arctan\sqrt{3}$ or $\cosh(\ln 3)$ should be simplified.

• Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

1. [16 Points] Find the **Interval** and **Radius** of Convergence for the following power series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n\ (5x+1)^n}{\sqrt{n}\cdot 9^n}$. Analyze carefully and with full justification.

2. [22 Points]

(a) Find the **MacLaurin series** representation for $f(x) = x^3 \arctan(7x)$. Simplify. **State** the Radius of Convergence for this series. Your answer should be in sigma notation $\displaystyle\sum_{n=0}^{\infty}$.

(b) Use Series to compute $\displaystyle\int \cos\left(x^2\right) - 1 + \frac{x^4}{2}\ dx$. Your answer should be in sigma notation $\displaystyle\sum_{n=2}^{\infty}$.

For parts (c) and (d), you do not need to find the Radius of Convergence. Justify all details.

(c) Find the **MacLaurin series** representation for $f(x) = \cosh x$, using any method.

(d) Demonstrate a **second, different** method/approach from part (c) above, to compute the MacLaurin Series for the same function, $f(x) = \cosh x$.

3. [12 Points] Use the MacLaurin Series representation for $f(x) = x^2 e^{-x^3}$ to

$$\textbf{Estimate} \int_0^1 x^2 e^{-x^3}\ dx \quad \text{with error less than } \frac{1}{50}.$$

Justify in words that your error is indeed less than $\frac{1}{50}$.

4. [24 Points] Find the **sum** for each of the following series. Simplify, if possible.

(a) $4 - \frac{4}{3} + \frac{4}{5} - \frac{4}{7} + \frac{4}{9} - \dots$ (b) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n\ \pi^{2n}}{9^n\ (2n+1)!}$ (c) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^{n+1}\ 2^{n+1}(\ln 9)^n}{n!}$

(d) $1 - e + \frac{e^2}{2!} - \frac{e^3}{3!} + \frac{e^4}{4!} - \frac{e^5}{5!} + \dots$ (e) $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \dots$

(f) $1 - \frac{\pi^2}{(4)2!} + \frac{\pi^4}{(16)4!} - \frac{\pi^6}{(64)6!} + \frac{\pi^8}{(256)8!} - \dots = 1 - \frac{\pi^2}{(2)^2 2!} + \frac{\pi^4}{(2)^4 4!} - \frac{\pi^6}{(2)^6 6!} + \frac{\pi^8}{(2)^8 8!} - \dots$

(g) $\displaystyle\sum_{n=0}^{\infty} \frac{e^6}{n!}(x-6)^n$ Your answer will involve x.

5. [14 Points] Volumes of Revolution

(a) Consider the same region bounded by $y = \arctan x$, $y = 2$, $x = 0$ and $x = 1$. Rotate this region about the *vertical* line $\boxed{x = -2}$. Set-up, **BUT DO NOT EVALUATE!!**, the integral to compute the volume of the resulting solid using the Cylindrical Shells Method. Sketch the solid, along with one of the approximating shells.

(b) Consider the different region bounded by $y = \ln x$, $y = 1$, and $x = 4$. Rotate this region about the vertical line $\boxed{x = 5}$. Set-up, **BUT DO NOT EVALUATE!!**, the integral to compute the volume of the resulting solid using the Cylindrical Shells Method. Sketch the solid, along with one of the approximating shells.

6. [12 Points] Consider the Parametric Curve given by $x = e^t + \dfrac{1}{1+e^t}$ and $y = 2\ln\left(1+e^t\right)$.

<u>**COMPUTE**</u> the **Arclength** of this parametric curve for $0 \le t \le \ln 3$.

Hint: $\dfrac{dx}{dt} = e^t - \dfrac{e^t}{(1+e^t)^2}$ and $\dfrac{dy}{dt} = \dfrac{2e^t}{(1+e^t)}$

****

OPTIONAL BONUS

Do not attempt these unless you are completely done with the rest of the exam.

****

OPTIONAL BONUS #1 Compute $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n x^n}{(2n+1)!}$ (Assume $x > 0$)

OPTIONAL BONUS #2 Compute $\displaystyle\sum_{n=0}^{\infty} \frac{(x+1)^{n+1}}{(n+2)!}$

OPTIONAL BONUS #3 Demonstrate a **third, different** method/approach than you used in parts 2(c) and 2(d) above, to compute the MacLaurin Series for the function, $f(x) = \cosh x$.