Name:_____

**Amherst College**

**DEPARTMENT OF MATHEMATICS**

**Math 121**

**Midterm Exam #3**

**December 2, 2011**

• This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or other aids are permitted.

• You need *not* simplify algebraically complicated answers. However, numerical answers such as $\sin\left(\frac{\pi}{6}\right)$, $4^{\frac{3}{2}}$, $e^{\ln 4}$, $\ln(e^7)$, $e^{3\ln 3}$, or $\cosh(\ln 3)$ should be simplified.

• Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

• If you actually read these directions, draw a smiley face at the bottom of this page!

| Problem | Score | Possible Points |
|---|---|---|
| 1 | | 12 |
| 2 | | 8 |
| 3 | | 12 |
| 4 | | 12 |
| 5 | | 16 |
| 6 | | 20 |
| 7 | | 20 |
| Total | | 100 |

**1.** [12 Points] Find the **Interval** and **Radius** of Convergence for the following power series. Analyze carefully and with full justification.

$$\sum_{n=1}^{\infty} \frac{(-1)^n (3x+2)^n}{n\ 5^{n+1}}$$

**2.** [8 Points] Find the **Taylor polynomial of degree 3** for $\mathbf{f(x)} = \cosh \mathbf{x}$ centered at $\mathbf{a} = \ln \mathbf{2}$.

**3.** [12 Points] Find the **MacLaurin series** representation for each of the following functions. State the Radius of Convergence for each series. Your answer should be in sigma notation $\sum_{n=0}^{\infty}$ .

(a) $f(x) = x^2 e^{-5x}$

(b) $f(x) = x^7 \sin(x^3)$

(c) $f(x) = \dfrac{1}{(1-x)^2}$

**4.** [12 Points] Use Power Series to estimate $\int_0^1 x^3 \ln(1+x^2)\, dx$ with error less than $\frac{1}{10}$.

Justify in words that your error is indeed less than $\frac{1}{10}$.

**5.** [16 Points] Find the **sum** for each of the following series.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n \ (\pi^2)^n}{9^n \ (2n)!}$

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n \ 9^n}{5^n \ n!}$

(c) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n \ \pi^{2n}}{4^{2n} \ (2n+1)!}$

(d) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} + \ldots$

**6.** [20 Points] Volumes of Revolution

(a) Consider the region bounded by $y = e^{2x}$, $x = 0$, and $y = 3$. Rotate this region about the horizontal line $\boxed{y = -1}$. Set-up, **BUT DO NOT EVALUATE!!**, the integral to compute the volume of the resulting solid using the Washer Method. Sketch the solid, along with one of the approximating washers.

(b) Consider the region bounded by $y = e^x$, $y = \ln x$, $x = 1$ and $x = 5$. Rotate this region about the vertical line $\boxed{x = 7}$. Set-up, **BUT DO NOT EVALUATE!!**, the integral to compute the volume of the resulting solid using the Cylindrical Shells Method. Sketch the solid, along with one of the approximating shells.

**6.** (Continued) Volumes of Revolution

(c) Consider the region bounded by $y = \arctan x$, $y = 0$, $x = 0$ and $x = 1$. Rotate this region about the $y$-axis. **COMPUTE** the volume of the resulting solid using the Cylindrical Shells Method. Sketch the solid, along with one of the approximating shells.

**7.** [20 Points] Consider the Parametric Curve represented by $x = t - e^t$ and $y = 1 - 4e^{\frac{t}{2}}$.

(a) **Compute** $\dfrac{dy}{dx}$ for this curve when $t = \ln 4$.

(b) **Compute** the **arclength** of this parametric curve for $0 \leq t \leq 1$.

(c) Set-up, **BUT DO NOT EVALUATE!!** the definite integral representing the **surface area** obtained by rotating this curve about the $x$**-axis**, for $0 \leq t \leq 1$.

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# OPTIONAL BONUS

Do not attempt these unless you are completely done with the rest of the exam.

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**OPTIONAL BONUS #1** Compute the **sum** $\sum_{n=0}^{\infty} \frac{n}{5^n}$

**OPTIONAL BONUS #2** Compute the **sum** $\sum_{n=0}^{\infty} \frac{n^3}{2^n n!}$

**OPTIONAL BONUS #3** Compute the MacLaurin Series for $f(x) = \frac{x}{(1-2x)^3}$ and state its Radius of Convergence.