Math 121 Exam #2 Mar 29-Mar 31

Due Sunday, March 31, 2024 in Gradescope by 11:59 pm

- This is **NOT** an Open Notes Exam. You can **NOT** access any materials, homeworks problems, lecture notes, etc. You may use one 5x7 Cheat Sheet.
- There is **NO** *Open Internet* access allowed. Do **NOT** use any online sources.
- You are not allowed to discuss these problems with anyone, including Math Fellows.
- Submit your final work in Gradescope in the Exam 2 entry.
- Please *show* all of your work and *justify* all of your answers. No Calculators.

1. [36 Points] Compute the following **Improper** integrals. Simplify all answers. Justify.

(a) $\displaystyle\int_0^e x^2 \ln\left(x^2\right)\, dx = \boxed{\frac{4e^3}{9}}$

(b) $\displaystyle\int_e^\infty \frac{\ln x}{x^2}\, dx = \int_e^\infty (\ln x)\, x^{-2}\, dx = \boxed{\frac{2}{e}}$

(c) $\displaystyle\int_{-\infty}^{-3} \frac{8-x}{x^2+2x+5}\, dx = \boxed{\infty}$

(d) $\displaystyle\int_{-4}^{-3} \frac{8-x}{x^2+2x-8}\, dx = \boxed{-\infty}$

2. [9 Points] Use the Integral Test to determine if $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+4n+7}$ **Converges** or **Diverges**.

Note: You do **not** have to check the 3 pre-conditions.

3. [32 Points] Determine whether each of the given series **Converges** or **Diverges**. Name any Convergence Test(s) you use, and justify all of your work.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{n^5+8}{8n^5+1}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(n+5)^8}{\ln(n+5)}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^8}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{\ln 5}{(n+5)^8} + \frac{(-1)^n \cdot 8}{5^{2n+1}}$

(e) $-1 - \frac{1}{2} - \frac{1}{3} - \frac{1}{4} - \frac{1}{5} - \dots$

4. [26 Points] Here you cannot choose series from this exam. Tip: *Keep the choices simple.* **DO NOT ACCESS YOUR NOTES OR ONLINE SOURCES!!**

(a) Create a Series that is **Divergent** by the n^{th} Term Divergence Test and also needs L'Hôpital's Rule to justify the Limit. Continue on to Prove it is Divergent.

(b) Create a Series that is **Convergent** by the Comparison Test. You cannot choose just a p-series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^p}$. Continue on to prove it is Convergent.

(c) Create a Series that is **Absolutely Convergent** by the Ratio Test. You cannot choose just a Geometric Series. Continue on to prove it is Absolutely Convergent.

(d) Create an Alternating Series that is **Convergent** by the Absolute Convergence Test. You cannot choose just an alternating p-series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^p}$. Continue on to prove it is Convergent.

5. [27 Points] Determine whether the given series is **Absolutely Convergent**, **Conditionally Convergent**, or **Divergent**. Name any Convergence Test(s) you use, and justify all of your work.

(a) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \left(\frac{n^5 + 5n + 8}{n^8 + 5} \right)$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \; n^5 \cdot n^n \cdot n!}{(2n+1)!}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{5n + 8}$