Math 121 Midterm Exam #2 March 23, 2018

• This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or other aids are permitted. Do not access any webpages during this exam.

• Numerical answers such as $\sin\left(\frac{\pi}{6}\right)$, $4^{\frac{3}{2}}$, $e^{\ln 4}$, $\ln(e^7)$, $e^{3\ln 3}$, $\sinh(\ln 3)$, or $\arctan(\sqrt{3})$ should be simplified.

• Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

1. [40 Points] Compute the following integrals. Justify your work.

(a) $\displaystyle\int_0^{e^4} \frac{1}{x\ [16+(\ln x)^2]}\ dx$

(b) $\displaystyle\int \frac{x^3+1}{x^2+1}\ dx$

(c) $\displaystyle\int_1^2 \frac{4}{x^2-6x+5}\ dx$

(d) $\displaystyle\int_4^\infty \frac{4}{x^2-6x+12}\ dx$

2. [10 Points] (a) Determine **and state** whether the following sequence **converges** or **diverges**. If it converges, compute its limit. Justify your answer. Do **not** just put down a number.

$$\left\{ \left(1-\frac{4}{n^2}\right)^{n^2} \right\}_{n=1}^{\infty}$$

(b) Determine **and state** whether the following series **converges** or **diverges**. Justify your answer.

$$\sum_{n=1}^{\infty} \left(1-\frac{4}{n^2}\right)^{n^2}$$

3. [8 Points] Find the **sum** of the following series (which does converge).

$$\sum_{n=1}^{\infty}(-1)^n\ \frac{7\cdot 3^{n+1}}{2^{4n-1}}$$

4. [18 Points] Determine whether each of the following series **converges** or **diverges**. Name any convergence test(s) you use, and justify all of your work.

(a) $\displaystyle\sum_{n=1}^{\infty} \sin^2\left(\frac{\pi n^4+1}{3n^4+5}\right)$

(b) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \, \frac{\sin^2(\pi n^4+1)}{3n^4+5}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2018} + \frac{1}{(2018)^n}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{2018}} + \frac{1}{(2018)^n}$

5. [24 Points] Determine whether the given series is **absolutely convergent**, **conditionally convergent**, or **diverges**. Name any convergence test(s) you use, and justify all of your work.

(a) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \, \frac{3n^5+n^2}{n^9+4}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \, (2n)!}{e^n \, (n^n) \, n!}$

(c) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \, \frac{1}{5n-4}$

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OPTIONAL BONUS

Do not attempt this unless you are completely done with the rest of the exam.

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OPTIONAL BONUS #1 Compute the sum of the following series

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}\,(n+1) + n\sqrt{n+1}}$$

OPTIONAL BONUS #2 Compute: $\displaystyle\int \frac{e^{2x}}{e^{8x}-1}\,dx$