Math 121 Midterm Exam #2 March 24, 2017

• This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or other aids are permitted. Do not access any webpages during this exam.

• Numerical answers such as $\sin\left(\frac{\pi}{6}\right)$, $4^{\frac{3}{2}}$, $e^{\ln 4}$, $\ln(e^7)$, $e^{3\ln 3}$, $\sinh(\ln 3)$, or $\arctan(\sqrt{3})$ should be simplified.

• Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

1. [40 Points] Compute the following integrals. If it diverges, justify your work.

(a) $\displaystyle\int \frac{x^3+x^2+10x+10}{x^2+9}\,dx$

(b) $\displaystyle\int_7^{\infty} \frac{1}{x^2-8x+19}\,dx$

(c) $\displaystyle\int_0^{e} \frac{\ln x}{\sqrt{x}}\,dx$

(d) $\displaystyle\int_7^{9} \frac{10}{x^2-8x-9}\,dx$

2. [10 Points] Determine **and state** whether the following sequence **converges** or **diverges**. If it converges, compute its limit. Justify your answer. Do **not** just put down a number.

$$\left\{ \left(\frac{n}{n+7}\right)^n \right\}_{n=1}^{\infty}$$

3. [10 Points] Find the **sum** of the following series (which does converge).

$$\sum_{n=1}^{\infty} (-1)^n \frac{5^{n+1}}{2^{3n-1}}$$

4. [15 Points] Determine whether each of the following series **converges** or **diverges**. Name any convergence test(s) you use, and justify all of your work.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{e^n}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{e}$

(c) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{\arctan n}{n^3+1}$

5. [25 Points] Determine whether the given series is **absolutely convergent**, **conditionally convergent**, or **diverges**. Name any convergence test(s) you use, and justify all of your work.

(a) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n^2+7}{3n^7+5}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n\ (2n)!\ n^6}{2^n\ (\ln n)\ (n^n)\ n!}$

(c) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n+7}$

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OPTIONAL BONUS

Do not attempt this unless you are completely done with the rest of the exam.

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OPTIONAL BONUS #1

Compute the sum of the following series

$$\sum_{n=2}^{\infty} \frac{e^{2n+2}-e^{2n}}{(e^{2n}+1)(e^{2n+2}+1)}$$

OPTIONAL BONUS #2

Prove the following statement:

If the series $\displaystyle\sum_{n=1}^{\infty} a_n$ converges, then $\displaystyle\lim_{n\to\infty} a_n = 0$

OPTIONAL BONUS #3

Compute the following integral $\displaystyle\int \frac{x^5+7x^3+x^2+13x+2}{x^4+6x^2+9}\,dx.$