Name:_____

Amherst College

DEPARTMENT OF MATHEMATICS

Math 121

Midterm Exam #2

March 25, 2016

• This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or other aids are permitted. Do not access any webpages during this exam.

• Numerical answers such as $\sin\left(\frac{\pi}{6}\right)$, $4^{\frac{3}{2}}$, $e^{\ln 4}$, $\ln(e^7)$, $e^{3\ln 3}$, $\sinh(\ln 3)$, or $\arctan(\sqrt{3})$ should be simplified.

• Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

Problem	Score	Possible Points
1		35
2		15
3		10
4		15
5		25
Total		100

1. [35 Points] Compute the following integral, or else show that it diverges.

(a) $\displaystyle\int_1^{\sqrt{3}} \frac{x^3+1}{x^2+1}\,dx$

1. (Continued) Compute each of the following integrals or show that it diverges.

(b) $\displaystyle\int_{-\infty}^{\infty} \frac{1}{x^2 - 10x + 28}\, dx$

(c) $\displaystyle\int_{0}^{1} \ln x\, dx$

1. (Continued) Compute the following integral, or else show that it diverges.

(d) $\displaystyle\int_1^7 \frac{15-x}{x^2-6x-7}\,dx$

2. [15 Points]

(a) Determine **and state** whether the following sequence **converges** or **diverges**. If it converges, compute its limit. Justify your answer. Do **not** just put down a number.

$$\left\{ \left(1 + \arcsin\left(\frac{1}{n}\right)\right)^n \right\}_{n=1}^{\infty}$$

(b) Determine **and state** whether the following series **converges** or **diverges**. Justify your answer.

$$\sum_{n=1}^{\infty} \left(1 + \arcsin\left(\frac{1}{n}\right)\right)^n$$

3. [10 Points] Find the **sum** of the following series (which does converge).

$$\sum_{n=1}^{\infty}(-1)^n \frac{6^{n+1}}{5^{3n-1}}$$

4. [15 Points] Determine whether each of the following series **converges** or **diverges**. Name any convergence test(s) you use, and justify all of your work.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{3}$

(b) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \, \frac{3 \, \cos^2(5n)}{n^5 + 7n}$

5. [25 Points] In each case determine whether the given series is **absolutely convergent**, **conditionally convergent**, or **diverges**. Name any convergence test(s) you use, and justify all of your work.

(a) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \, \frac{n^2+7}{n^7+2}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \, (2n)! \, \ln n}{n^n \cdot \, n! \cdot \, n^6}$

5. (Continued) Determine whether the given series is **absolutely convergent**, **conditionally convergent**, or **diverges**. Name any convergence test(s) you use, and justify all of your work.

(c) $\displaystyle\sum_{n=2}^{\infty} (-1)^{n+1} \, \frac{n}{n^2 - 1}$

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OPTIONAL BONUS

Do not attempt this unless you are completely done with the rest of the exam.

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OPTIONAL BONUS #1 We have seen that the harmonic series is a diverent series whose terms do indeed approach zero. Show that the following series $\sum_{n=1}^{\infty} \ln\left(1+\frac{1}{n}\right)$ is another series with this property.

OPTIONAL BONUS #2 Compute the following integral $\int \ln(x^2 - x + 2)\, dx$.