Math 121 Midterm Exam #2 March 27, 2015

• This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or other aids are permitted. Do not access any webpages during this exam.

• You need *not* simplify algebraically complicated answers. However, numerical answers such as $\sin\left(\frac{\pi}{6}\right)$, $4^{\frac{3}{2}}$, $e^{\ln 4}$, $\ln(e^7)$, $e^{3\ln 3}$, $\sinh(\ln 3)$, or $\arctan(\sqrt{3})$ should be simplified.

• Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

1. [30 Points] Compute the following integral.

(a) $\displaystyle\int \frac{x^4+3x^3+7x^2+22x+7}{x^3+7x}\,dx$

(b) $\displaystyle\int_0^{\frac{1}{2}} \frac{1}{x\,\ln x}\,dx$

(c) $\displaystyle\int_8^{\infty} \frac{1}{x^2-10x+28}\,dx$

2. [10 Points] Determine **and state** whether the following sequence **converges** or **diverges**. If it converges, compute its limit. Justify your answer. Do **not** just put down a number.

$$\left\{\left(\frac{n}{n+5}\right)^n\right\}_{n=1}^{\infty}$$

3. [10 Points] Find the **sum** of the following series (which does converge).

$$\sum_{n=1}^{\infty}(-1)^n\,\frac{5^{n+1}}{3^{2n-1}}$$

4. [10 Points] Use the **Integral Test** to **determine** and **state** whether the series $\displaystyle\sum_{n=1}^{\infty}\frac{n}{e^{3n}}$ converges or diverges. Justify all of your work.

5. [15 Points] Determine whether each of the following series **converges** or **diverges**. Name any convergence test(s) you use, and justify all of your work.

(a) $\displaystyle\sum_{n=1}^{\infty} n \sin\left(\frac{1}{n}\right)$

(b) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{3}{n^3} + \frac{\sin^2(3n)}{3^n}\right)$

6. [25 Points] In each case determine whether the given series is **absolutely convergent**, **conditionally convergent**, or **diverges**. Name any convergence test(s) you use, and justify all of your work.

(a) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \, \frac{n^2+7}{n^7+2}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n (2n)!}{e^{2n} \cdot \; n! \cdot \; n^n}$

(c) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \, \frac{1}{\sqrt{n}+4}$

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OPTIONAL BONUS

Do not attempt these unless you are completely done with the rest of the exam.

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OPTIONAL BONUS #1 Compute the following integral $\displaystyle\int \frac{x^3+8}{(x^2-2x+3)^2} \, dx.$

OPTIONAL BONUS #2 Prove that the sequence $\left\{\frac{2^n\ n!}{n^n}\right\}_{n=1}^{\infty}$ converges and find its limit.