Math 121 Midterm Exam #2 March 28, 2014

• This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or other aids are permitted. Do not access any webpages during this exam.

• You need *not* simplify algebraically complicated answers. However, numerical answers such as $\sin\left(\frac{\pi}{6}\right)$, $4^{\frac{3}{2}}$, $e^{\ln 4}$, $\ln(e^7)$, $e^{3\ln 3}$, $\sinh(\ln 3)$, or $\arctan(\sqrt{3})$ should be simplified.

• Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

• If you actually read these directions, draw a smiley face at the bottom of the page.

1. [40 Points] Compute each of the following integrals, or else show that it diverges.

(a) $\displaystyle\int_3^\infty \frac{1}{x^2-4x+7}\,dx$

(b) $\displaystyle\int_0^1 \frac{1}{x-5}-\frac{1}{x-1}\,dx$

(c) $\displaystyle\int_{-\infty}^\infty \frac{x^2}{9+x^6}\,dx$

(d) $\displaystyle\int \frac{x^4+4x^2+x+4}{x^3+4x}\,dx$

2. [10 Points] Determine **and state** whether the following sequence **converges** or **diverges**. If it converges, compute its limit. Justify your answer. Do **not** just put down a number.

$$\left\{\left(\frac{n}{n+1}\right)^n\right\}_{n=1}^\infty$$

3. [10 Points] Find the **sum** of the following series (which does converge).

$$\sum_{n=1}^\infty \frac{(-1)^n\ 4^{n+1}}{3^{3n-1}}$$

4. [20 Points] Determine whether each of the following series **converges** or **diverges**. Name any convergence test(s) you use, and justify all of your work.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{\arctan(n+2014)}{n^{2014}+2014}$

(b) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{2014} + \frac{1}{(2014)^n}\right)$

(c) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{6}{n^6} + \frac{n^6}{6^n}\right)$

5. [20 Points] In each case determine whether the given series is **absolutely convergent**, **conditionally convergent**, or **diverges**. Name any convergence test(s) you use, and justify all of your work.

(a) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{\sqrt{n}+7}{n^7+7n+7}$

(b) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \frac{(2n)!}{4^n\ n!\ n^n}$

(c) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{1}{7n-4}$

****

OPTIONAL BONUS

Do not attempt these unless you are completely done with the rest of the exam.

****

OPTIONAL BONUS #1 Compute the sum of the following series:

1. $\displaystyle\sum_{n=1}^{\infty} \ln\left(1 + \frac{1}{n^2+2n}\right)$

OPTIONAL BONUS #2 Compute the following integral:

$\displaystyle\int \frac{\arctan x}{x^6}\, dx$