Math 121 Midterm Exam #2 March 29, 2013

• This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or other aids are permitted. Do not access any webpages during this exam.

• You need *not* simplify algebraically complicated answers. However, numerical answers such as $\sin\left(\frac{\pi}{6}\right)$, $4^{\frac{3}{2}}$, $e^{\ln 4}$, $\ln(e^7)$, $e^{3\ln 3}$, or $\arctan(\sqrt{3})$ should be simplified.

• Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

• If you actually read these directions, draw a smiley face at the bottom of the page.

1. [40 Points] Compute the following integral(s), or else show that it diverges.

(a) $\displaystyle\int_1^8 \frac{1}{(x-5)^3}\,dx$

(b) $\displaystyle\int \frac{x^4+4x^2+x+4}{x^3+4x}\,dx$

(c) $\displaystyle\int_3^\infty \frac{1}{x-2}-\frac{1}{x+5}\,dx$

(d) $\displaystyle\int_7^\infty \frac{1}{x^2-8x+19}\,dx$

2. [10 Points] Determine **and state** whether the following sequence **converges** or **diverges**. If it converges, compute its limit. Justify your answer. Do **not** just put down a number.

$$\left\{\left(\frac{n}{n+1}\right)^n\right\}_{n=1}^{\infty}$$

3. [10 Points] Find the **sum** of the following series (which does converge):

$$\sum_{n=1}^{\infty}\frac{(-1)^n\ 3^{n+2}}{2^{3n-1}}$$

4. [18 Points] Determine whether each of the following series **converges** or **diverges**. Name any convergence test(s) you use, and justify all of your work.

(a) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{7}{n^7} + \frac{1}{7^n}\right)$

(b) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n}{n+1}\right)^n$ (Hint: See #2)

(c) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{\arctan n}{n^{2013} + 2013}$

5. [22 Points] In each case determine whether the given series is **absolutely convergent**, **conditionally convergent**, or **diverges**. Name any convergence test(s) you use, and justify all of your work.

(a) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n^3 + 7n^2 + 5}{n^8 + n\sqrt{n} + 4}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1} n^n}{e^{6n}\, n!}$

(c) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt{n} + 3}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n^2}$

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OPTIONAL BONUS

Do not attempt these unless you are completely done with the rest of the exam.

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OPTIONAL BONUS #1 Compute the sum of the following series:

1. $\displaystyle\sum_{n=1}^{\infty} \frac{e^{2n+2} - e^{2n}}{(e^{2n} + 1)(e^{2n+2} + 1)}$

OPTIONAL BONUS #2 Compute the following integral:

2. $\displaystyle\int \frac{x^5 + 7x^3 + x^2 + 13x + 2}{x^4 + 6x^2 + 9}\, dx$