Math 12 Midterm Exam #2 (Compact Version) March 28, 2012

1. [34 Points] Compute the following integral, or else show that it diverges.

(a) $\displaystyle\int_0^2 \frac{1}{(x-1)^2}\,dx$

(b) $\displaystyle\int \frac{x^4+x^3+4x^2+5x+4}{x^3+4x}\,dx$

(c) $\displaystyle\int_3^\infty \frac{1}{x^2-4x+7}\,dx$

2. [10 Points] Determine whether the following sequence **converges** or **diverges**. If it converges, compute its limit. Justify your answer. Do **not** just put down a number.

$$\left\{\left(\frac{n+1}{n}\right)^n\right\}_{n=1}^{\infty}$$

3. [10 Points] Find the **sum** of the following series (which does converge):

$$\sum_{n=1}^{\infty} \frac{(-1)^n\ 4^{n+1}}{3^{3n-1}}$$

4. [10 Points] Use the **Integral Test** to prove that the Harmonic Series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$ **diverges**. Justify all of your work.

5. [15 Points] Determine whether each of the following series **converges** or **diverges**. Name any convergence test(s) you use, and justify all of your work.

(a) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{n^6}+\frac{1}{6^n}\right)$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{7n-1}{8n+1}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{\arctan n}{n^3+1}$

6. [21 Points] In each case determine whether the given series is **absolutely convergent**, **conditionally convergent**, or **diverges**. Name any convergence test(s) you use, and justify all of your work.

(a) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \, \frac{n^4 + 5n^2 + 9}{n^7 + 3n - 1}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1} n^n}{e^{6n} \, n!}$

(c) $\displaystyle\sum_{n=9}^{\infty} (-1)^n \, \frac{1}{n - 8}$

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OPTIONAL BONUS

Do not attempt these unless you are completely done with the rest of the exam.

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OPTIONAL BONUS #1 Compute the sum of the following series:

1. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}(n+1) + n\sqrt{n+1}}$

OPTIONAL BONUS #2 Compute the following integral:

2. $\displaystyle\int \frac{\arctan x}{x^6} \, dx$