## Math 12     Midterm Exam #2 (Compact Version)     March 30, 2011

• This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or other aids are permitted.

• You need *not* simplify algebraically complicated answers. However, numerical answers such as $\sin\left(\dfrac{\pi}{6}\right)$, $4^{\frac{3}{2}}$, $e^{\ln 4}$, $\ln(e^7)$, or $e^{3\ln 3}$ should be simplified.

• Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

**1.** [40 Points]   Compute the following integral, or else show that it diverges.

(a) $\displaystyle\int_1^9 \frac{1}{(x-7)^2}\,dx$

(b) $\displaystyle\int \frac{4x+1}{x^2-3x-10}\,dx$

(c) $\displaystyle\int \frac{x^4+x^2+x+1}{x^3+x}\,dx$

(d) $\displaystyle\int_9^\infty \frac{1}{x^2-8x+41}\,dx$

**2.** [8 Points]   Determine whether the following sequence **converges** or **diverges**. If it converges, compute its limit. Justify your answer. Do not just put down a number.

$$\left\{\left(\frac{n+1}{n}\right)^n\right\}_{n=1}^\infty$$

**3.** [8 Points]   Find the **sum** of the following series (which does converge):

$$\sum_{n=1}^\infty \frac{(-1)^n\,3^{n+2}}{2^{4n-1}}$$

**4.** [20 Points]   Determine whether each of the following series **converges** or **diverges**. Name any convergence test(s) you use, and justify all of your work.

(a) $\displaystyle\sum_{n=1}^\infty \frac{n\,\sin^2 n}{8n^2\sqrt{n}+n+7}$

(b) $\displaystyle\sum_{n=1}^\infty e^{\frac{\sin n}{n}}$

(c) $\displaystyle\sum_{n=2}^\infty \frac{1}{\ln n}$

(d) $\displaystyle\sum_{n=1}^\infty \frac{n^n}{e^{2n}n!}$

**5.** [24 Points]  In each case determine whether the given series is **absolutely convergent**, **conditionally convergent**, or **diverges**. Name any convergence test(s) you use, and justify all of your work.

(a) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{n^2+1}$

(b) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n^2}{7^n}$

(c) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n^6 + 5n^2 + 826}{n^9 + 7n^3 + 2011}$

(d) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{1}{3^n}$

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# OPTIONAL BONUS
Do not attempt these unless you are completely done with the rest of the exam.
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**OPTIONAL BONUS #1**  Compute the sum of the following series:

1. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+3n}$

**OPTIONAL BONUS #2**  Determine whether the following series converges or diverges.

2. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n^{4n}}{n^7 (n!)^2 e^{8n} (2n)!}$

**OPTIONAL BONUS #3**  Compute the following integral:

3. $\displaystyle\int \frac{\cos x}{\sin^3 x - 1}\, dx$

**OPTIONAL BONUS #4**  Compute the following integral:

4. $\displaystyle\int \frac{x^5 + x^4 + 19x^3 + 18x^3 + 18x^2 + 81x + 81}{x^4 + 18x^2 + 81}\, dx$