Math 121 Exam #2 Nov 2-Nov 5

Due Sunday, November 5, 2023 in Gradescope by 11:59 pm

• This is **NOT** an Open Notes Exam. You can **NOT** access any materials, homeworks problems, lecture notes, etc. You may use one 5x7 Cheat Sheet.

• There is **NO** *Open Internet* access allowed. Do **NOT** use any online sources.

• You are not allowed to discuss these problems with anyone, including Math Fellows.

• Submit your final work in Gradescope in the Exam 2 entry.

• Please *show* all of your work and *justify* all of your answers. No Calculators.

1. [36 Points] Compute the following **Improper** integrals. Simplify all answers. Justify.

(a) $\displaystyle\int_0^e x^2 \ln\left(x^2\right)\ dx = \boxed{\dfrac{4e^3}{9}}$

(b) $\displaystyle\int_e^\infty \dfrac{\ln x}{x^2}\ dx = \int_e^\infty (\ln x)\ x^{-2}\ dx = \boxed{\dfrac{2}{e}}$

(c) $\displaystyle\int_{-\infty}^{-3} \dfrac{8-x}{x^2+2x+5}\ dx = \boxed{\infty}$

(d) $\displaystyle\int_{-4}^{-3} \dfrac{8-x}{x^2+2x-8}\ dx = \boxed{-\infty}$

2. [9 Points] Use the Integral Test to determine if $\displaystyle\sum_{n=1}^\infty \dfrac{1}{n^2+4n+7}$ **Converges** or **Diverges**.

Note: You do **not** have to check the 3 pre-conditions.

3. [30 Points] Determine whether each of the given series **Converges** or **Diverges**. Name any Convergence Test(s) you use, and justify all of your work.

(a) $\displaystyle\sum_{n=1}^\infty n^5 + n^4 + n^3 + n^2 + n + 1$

(b) $\displaystyle\sum_{n=1}^\infty \dfrac{(n+5)^8}{\ln(n+5)}$

(c) $\displaystyle\sum_{n=1}^\infty \dfrac{(-1)^n}{n^8}$

(d) $\displaystyle\sum_{n=1}^\infty \dfrac{\ln 5}{(n+5)^8} + \dfrac{(-1)^n \cdot 8}{5^{2n+1}}$

(e) $-1 - \dfrac{1}{2} - \dfrac{1}{3} - \dfrac{1}{4} - \dfrac{1}{5} - \cdots$

4. [8 Points] Use the Absolute Convergence Test to show that $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^5 + n^4 + n^3 + n^2 + n + 1}$

Converges.

5. [10 Points] Here you cannot choose series from this exam. Tip: Keep the choices simple.

(a) Create a Series that is Divergent by the nth Term Divergence Test and also needs L'Hôpital's Rule to justify the Limit. Continue on to Prove it is Divergent.

(b) Create a Series that is Convergent by the Comparison Test. You cannot choose just a p-series. Continue on to prove it is Convergent.

6. [27 Points] Determine whether the given series is **Absolutely Convergent**, **Conditionally Convergent**, or **Divergent** or follow the description. Name any Convergence Test(s) you use, and justify all of your work.

(a) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \left(\frac{n^5 + 5n + 8}{n^8 + 5} \right)$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \, n^5 \cdot n^n \cdot n!}{(2n+1)!}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{5n + 8}$