Math 121 Midterm Exam #2 October 27, 2021

• This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or other aids are permitted. Do not access any webpages during this exam.

• Numerical answers such as $\sin\left(\frac{\pi}{6}\right)$, $4^{\frac{3}{2}}$, $e^{\ln 4}$, $\ln(e^7)$, $e^{3\ln 3}$, $\sinh(\ln 3)$, or $\arctan(\sqrt{3})$ should be simplified.

• Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

1. [36 Points] Compute the following **Improper** Integrals. Justify your work.

(a) $\displaystyle\int_0^{e^4} \frac{1}{x\ [16 + (\ln x)^2]}\, dx$

(b) $\displaystyle\int_{-\infty}^{0} \frac{1}{x^2 + 2x + 4}\, dx$

(c) $\displaystyle\int_0^{e} \ln x\ dx$

(d) $\displaystyle\int_1^{\sqrt{3}} \frac{x^4 - x^3 + 3x^2 - x + 2}{x^3 - x^2 + 3x - 3}\, dx = \int_1^{\sqrt{3}} \frac{x^4 - x^3 + 3x^2 - x + 2}{(x-1)(x^2+3)}\, dx$

You can use this **free** Partial Fractions fact:

$$\frac{2x+2}{(x-1)(x^2+3)} = \frac{1}{x-1} + \frac{1-x}{x^2+3}$$

Finish computing all finite values here.

2. [12 Points] Name any convergence test(s) you use, and justify all of your work.

(a) Determine **and state** whether the following sequence **converges** or **diverges**.

$$\left\{ n \sin\left(\frac{1}{n}\right) \right\}_{n=1}^{\infty}$$

(b) Determine **and state** whether the following series **converges** or **diverges**.

$$\sum_{n=1}^{\infty} n \sin\left(\frac{1}{n}\right)$$

3. [12 Points] Name any convergence test(s) you use, and justify all of your work.

Use the Absolute Convergence Test to prove that the series $\sum_{n=1}^{\infty}(-1)^n \frac{\cos^2 n}{n^7+6}$ Converges.

4. [40 Points] For each part, you do not need to choose complicated Infinite Series. If you choose to use the Ratio Test, then you can only use the Ratio Test for **at most** one part (a), (b), or (c).

(a) Give an example of an **Alternating** Series which is **Absolutely Convergent**. You **cannot** choose just $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^p}$ or just a Geometric Series or the series in #3 above. Continue on to **JUSTIFY** that this series is Absolutely Convergent.

(b) Give an example of an Infinite Series which is **Divergent**. You **cannot** choose just $\sum_{n=1}^{\infty}$ constant or a p-series or a Geometric Series or the series in #2 above. Continue on to **JUSTIFY** that this series is Divergent.

(c) Give an example of an **Alternating** Series which is **Conditionally Convergent**. You **cannot** choose just $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^p}$. Continue on to **JUSTIFY** that this series is Conditionally Convergent.

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OPTIONAL BONUS

Do not attempt this unless you are completely done with the rest of the exam.

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OPTIONAL BONUS #1 Prove that the sequence $\left\{\frac{2^n\ n!}{n^n}\right\}_{n=1}^{\infty}$ converges.