Math 121 Midterm Exam #2 October 31, 2018

• This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or other aids are permitted. Do not access any webpages during this exam.

• Numerical answers such as $\sin\left(\frac{\pi}{6}\right)$, $4^{\frac{3}{2}}$, $e^{\ln 4}$, $\ln(e^7)$, $e^{3\ln 3}$, $\sinh(\ln 3)$, or $\arctan(\sqrt{3})$ should be simplified.

• Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

1. [40 Points] Compute the following integrals. Justify your work.

(a) $\displaystyle\int_0^{e^3} \frac{1}{x\ [9+(\ln x)^2]}\ dx$

(b) $\displaystyle\int_0^1 x \ln x\ dx$

(c) $\displaystyle\int_7^9 \frac{10}{x^2-8x-9}\ dx$

(d) $\displaystyle\int_7^\infty \frac{10}{x^2-8x+19}\ dx$

2. [10 Points] (a) Determine **and state** whether the following sequence **converges** or **diverges**. If it converges, compute its limit. Justify your answer. Do **not** just put down a number.

$$\left\{ \left(\frac{n+1}{n}\right)^n \right\}_{n=1}^{\infty}$$

(b) Determine **and state** whether the following series **converges** or **diverges**. Justify your answer.

$$\sum_{n=1}^{\infty} \left(\frac{n+1}{n}\right)^n$$

3. [8 Points] Find the **sum** of the following series (which does converge).

$$\sum_{n=1}^{\infty} (-1)^n\ \frac{3^{2n-1}}{4^{2n+1}}$$

4. [18 Points] Determine whether each of the following series **converges** or **diverges**. Name any convergence test(s) you use, and justify all of your work.

(a) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \, \frac{\sin^2(n^3+1)}{n^3+1}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{e} + \frac{1}{e^n}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^e} + \frac{1}{e^n}$

5. [24 Points] Determine whether the given series is **absolutely convergent**, **conditionally convergent**, or **diverges**. Name any convergence test(s) you use, and justify all of your work.

(a) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \, \frac{n^2+7n}{n^9+2}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \, (2n)! \, n^6}{2^n \, (n^n) \, n!}$

(c) $\displaystyle\sum_{n=8}^{\infty} (-1)^{n+1} \, \frac{1}{n-7}$

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OPTIONAL BONUS

Do not attempt this unless you are completely done with the rest of the exam.

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OPTIONAL BONUS #1 Prove that the sequence $\displaystyle\left\{ \frac{2^n \, n!}{n^n} \right\}_{n=1}^{\infty}$ converges.