Math 121 **Midterm Exam #2** **November 2, 2016**

• This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or other aids are permitted. Do not access any webpages during this exam.

• Numerical answers such as $\sin\left(\frac{\pi}{6}\right)$, $4^{\frac{3}{2}}$, $e^{\ln 4}$, $\ln(e^7)$, $e^{3\ln 3}$, $\sinh(\ln 3)$, or $\arctan(\sqrt{3})$ should be simplified.

• Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

1. [40 Points] Compute the following integral, or else show that it diverges.

(a) $\displaystyle\int_0^{\sqrt{3}} \frac{x^3+7x+1}{x^2+1}\,dx$

(b) $\displaystyle\int_5^{\infty} \frac{1}{x^2-6x+13}\,dx$

(c) $\displaystyle\int_0^1 \frac{\ln x}{\sqrt{x}}\,dx$

(d) $\displaystyle\int_1^2 \frac{1}{x^2-x}\,dx$

2. [10 Points] Determine **and state** whether the following sequence **converges** or **diverges**. If it converges, compute its limit. Justify your answer. Do **not** just put down a number.

$$\left\{ \left(1+\ln\left(1+\frac{5}{n}\right)\right)^n \right\}_{n=1}^{\infty}$$

3. [10 Points] Find the **sum** of the following series (which does converge).

$$\sum_{n=1}^{\infty}(-1)^n \frac{4^{2n+1}}{3^{3n-1}}$$

4. [15 Points] Determine whether each of the following series **converges** or **diverges**. Name any convergence test(s) you use, and justify all of your work.

(a) $\displaystyle\sum_{n=2}^{\infty} \frac{n^2}{\ln n}$

(b) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \, \frac{\sin^2(5n)}{n^5+3}$

5. [25 Points] In each case determine whether the given series is **absolutely convergent**, **conditionally convergent**, or **diverges**. Name any convergence test(s) you use, and justify all of your work.

(a) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \, \frac{3n^5+6n^3}{n^9+4}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \, (n!)^2 \, 2^{4n} \, n^n}{(3n)! \, \ln n}$

(c) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \, \frac{1}{\sqrt{n}+1}$

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OPTIONAL BONUS

Do not attempt this unless you are completely done with the rest of the exam.

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OPTIONAL BONUS #1 Compute the sum of the following series

$$\sum_{n=2}^{\infty} \frac{e^{2n+2}-e^{2n}}{(e^{2n}+1)(e^{2n+2}+1)}$$

OPTIONAL BONUS #2 Compute the following integral $\displaystyle\int \frac{x^5+7x^3+x^2+13x+2}{x^4+6x^2+9}\,dx.$