

Name: _____

Amherst College
DEPARTMENT OF MATHEMATICS
Math 121
Midterm Exam #2
October 29, 2014

- This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or other aids are permitted. Do not access any webpages during this exam.

- You need *not* simplify algebraically complicated answers. However, numerical answers such as $\sin\left(\frac{\pi}{6}\right)$, $4^{\frac{3}{2}}$, $e^{\ln 4}$, $\ln(e^7)$, $e^{3\ln 3}$, $\sinh(\ln 3)$, or $\arctan(\sqrt{3})$ should be simplified.

- Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

Problem	Score	Possible Points
1		40
2		10
3		10
4		14
5		26
Total		100

1. [40 Points] Compute the following integral.

(a) $\int \frac{x^4 + 3x^3 + 7x^2 + 22x + 7}{x^3 + 7x} dx$

1. (Continued) Compute each of the following integrals or show that it diverges.

(b) $\int_0^{\frac{1}{2}} \frac{1}{x \ln x} dx$

(c) $\int_8^{\infty} \frac{1}{x^2 - 10x + 28} dx$

1. (Continued) Compute the following integral or show that it diverges.

(d) $\int_2^{\infty} xe^{-3x} dx$

2. [10 Points] Determine **and state** whether the following sequence **converges** or **diverges**. If it converges, compute its limit. Justify your answer. Do **not** just put down a number.

$$\left\{ \left(\frac{n}{n+3} \right)^n \right\}_{n=1}^{\infty}$$

3. [10 Points] Find the **sum** of the following series (which does converge).

$$\sum_{n=1}^{\infty} (-1)^n \frac{(7) \cdot 3^{n+1}}{2^{4n-1}}$$

4. [14 Points] Determine whether each of the following series **converges** or **diverges**. Name any convergence test(s) you use, and justify all of your work.

(a)
$$\sum_{n=1}^{\infty} \ln(3n+2) - \ln(n+1)$$

(b)
$$\sum_{n=1}^{\infty} \left(\frac{3}{n^3} + \frac{n^3}{3^n} \right)$$

5. [26 Points] In each case determine whether the given series is **absolutely convergent**, **conditionally convergent**, or **diverges**. Name any convergence test(s) you use, and justify all of your work.

(a)
$$\sum_{n=1}^{\infty} (-1)^n \frac{\sqrt{n} + 3}{n^3 + 3n}$$

(b)
$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{(\ln n)(2n)!}{e^n \cdot n! \cdot n^n}$$

5. (Continued) Determine whether the given series is **absolutely convergent**, **conditionally convergent**, or **diverges**. Name any convergence test(s) you use, and justify all of your work.

(c) $\sum_{n=1}^{\infty} (-1)^n \frac{1}{5n-4}$

OPTIONAL BONUS

Do not attempt these unless you are completely done with the rest of the exam.

OPTIONAL BONUS #1 Compute the following integral $\int \frac{x^3 + 8}{(x^2 - 2x + 3)^2} dx$.

OPTIONAL BONUS #2 Prove that the sequence $\left\{ \frac{2^n n!}{n^n} \right\}_{n=1}^{\infty}$ converges and find its limit.