Math 121 Midterm Exam #2 October 30, 2013

• This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or other aids are permitted. Do not access any webpages during this exam.

• You need *not* simplify algebraically complicated answers. However, numerical answers such as $\sin\left(\frac{\pi}{6}\right)$, $4^{\frac{3}{2}}$, $e^{\ln 4}$, $\ln(e^7)$, $e^{3\ln 3}$, $\sinh(\ln 3)$, or $\arctan(\sqrt{3})$ should be simplified.

• Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

• If you actually read these directions, draw a smiley face at the bottom of the page.

1. [34 Points] Compute each of the following integrals, or else show that it diverges.

(a) $\displaystyle\int_7^\infty \frac{1}{x^2-6x+25}\,dx$

(b) $\displaystyle\int_{-\infty}^\infty \frac{x}{(x^2+4)^{\frac{3}{2}}}\,dx$

(c) $\displaystyle\int_0^1 \frac{1}{x}\,dx$

(d) $\displaystyle\int \frac{x^4+x^3+5x^2+3x+2}{x^3+x^2+3x+3}\,dx = \int \frac{x^4+x^3+5x^2+3x+2}{(x+1)(x^2+3)}\,dx$

2. [10 Points] Determine **and state** whether the following sequence **converges** or **diverges**. If it converges, compute its limit. Justify your answer. Do **not** just put down a number.

$$\left\{\left(\frac{n}{n+1}\right)^n\right\}_{n=1}^\infty$$

3. [10 Points] Find the **sum** of the following series (which does converge):

$$\sum_{n=1}^\infty \frac{(-1)^n\ 4^{n+1}}{3^{3n-1}}$$

4. [10 Points] Use the **Integral Test** to prove that the Harmonic Series $\displaystyle\sum_{n=1}^\infty \frac{1}{n}$ **diverges**. Justify all of your work.

5. [15 Points] Determine whether each of the following series **converges** or **diverges**. Name any convergence test(s) you use, and justify all of your work.

(a) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{5}{n^5} + \frac{1}{5^n} \right)$

(b) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{2013} + \frac{1}{(2013)^n} \right)$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{\arctan(n+2013)}{n^{2013}+2013}$

6. [21 Points] In each case determine whether the given series is **absolutely convergent**, **conditionally convergent**, or **diverges**. Name any convergence test(s) you use, and justify all of your work.

(a) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \, \frac{n^2+7}{n^{\frac{7}{2}}+n+7}$

(b) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \, \frac{n! \, 3^n}{e^n \, n^n}$

(c) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \, \frac{1}{7n-3}$

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OPTIONAL BONUS

Do not attempt this unless you are completely done with the rest of the exam.

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OPTIONAL BONUS #1 Compute the sum of the following series:

1. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}(n+1)+n\sqrt{n+1}}$