Midterm Exam #2

October 31, 2012

1. [40 Points] Compute the following integrals, or else show that they diverge.

(a) $\displaystyle\int_1^9 \frac{1}{(x-7)^3}\,dx$

(b) $\displaystyle\int \frac{x^4+4x^2+x+4}{x^3+4x}\,dx$

(c) $\displaystyle\int_2^\infty \frac{1}{x-1}-\frac{1}{x+3}\,dx$

(d) $\displaystyle\int_7^\infty \frac{1}{x^2-8x+19}\,dx$

2. [10 Points] Determine **and state** whether the following sequence **converges** or **diverges**. If it converges, compute its limit. Justify your answer. Do **not** just put down a number.

$$\left\{\left(\frac{n+3}{n}\right)^n\right\}_{n=1}^{\infty}$$

3. [10 Points] Find the **sum** of the following series (which does converge):

$$\sum_{n=1}^{\infty}\frac{(-1)^n\ 4^{n+1}}{3^{3n-1}}$$

4. [18 Points] Determine whether each of the following series **converges** or **diverges**. Name any convergence test(s) you use, and justify all of your work.

(a) $\displaystyle\sum_{n=1}^{\infty}\left(\frac{1}{n^6}+\frac{1}{6^n}\right)$

(b) $\displaystyle\sum_{n=1}^{\infty}\left(\frac{n+3}{n}\right)^n$ (Hint: See #2)

(c) $\displaystyle\sum_{n=1}^{\infty}(-1)^n\,\frac{\arctan n}{n^{1.01}+2012}$

5. [22 Points] In each case determine whether the given series is **absolutely convergent**, **conditionally convergent**, or **diverges**. Name any convergence test(s) you use, and justify all of your work.

(a) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \, \frac{n^4 + 5n^2 + 9}{n^7 + 3n - 1}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1} n^n}{e^{6n}\, n!}$

(c) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \, \frac{n}{n\sqrt{n} + 1}$

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OPTIONAL BONUS

Do not attempt these unless you are completely done with the rest of the exam.

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OPTIONAL BONUS #1 Compute the sum of the following series:

1. $\displaystyle\sum_{n=1}^{\infty} \frac{e^{2n+2} - e^{2n}}{(e^{2n} + 1)(e^{2n+2} + 1)}$

OPTIONAL BONUS #2 Compute the following integral:

2. $\displaystyle\int \frac{x^5 + 7x^3 + x^2 + 13x + 2}{x^4 + 6x^2 + 9}\, dx$