

Name: _____

Amherst College
DEPARTMENT OF MATHEMATICS
Math 121
Midterm Exam #2
October 28, 2011

- This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or other aids are permitted. Do not access any webpages during this exam.

- You need *not* simplify algebraically complicated answers. However, numerical answers such as $\sin\left(\frac{\pi}{6}\right)$, $4^{\frac{3}{2}}$, $e^{\ln 4}$, $\ln(e^7)$, or $e^{3\ln 3}$ should be simplified.

- Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

Problem	Score	Possible Points
1		40
2		8
3		8
4		24
5		20
Total		100

1. [40 Points] Compute the following integral, or else show that it diverges.

(a) $\int_0^2 \frac{1}{(x-1)^2} dx$

1. (Continued) Compute the following integral, or else show that it diverges.

(b) $\int_{-\infty}^{\infty} \frac{1}{1+x^2} dx$

1. (Continued) Compute the following integral, or else show that it diverges.

(c) $\int \frac{x^4 + x^3 + 4x^2 + 5x + 4}{x^3 + 4x} dx$

1. (Continued) Compute the following integral, or else show that it diverges.

(d) $\int_3^{\infty} \frac{1}{x^2 - 4x + 7} dx$

2. [8 Points] Determine whether the following sequence **converges** or **diverges**. If it converges, compute its limit. Justify your answer. Do **not** just put down a number.

$$\left\{ \left(\frac{n+1}{n} \right)^n \right\}_{n=1}^{\infty}$$

3. [8 Points] Find the **sum** of the following series (which does converge):

$$\sum_{n=1}^{\infty} \frac{(-1)^n 4^{n+1}}{3^{3n-1}}$$

4. [24 Points] Determine whether each of the following series **converges** or **diverges**. Name any convergence test(s) you use, and justify all of your work.

(a)
$$\sum_{n=1}^{\infty} \frac{n^2 + 7}{n^{\frac{7}{2}} + n + 7}$$

(b)
$$\sum_{n=1}^{\infty} \frac{7n - 1}{8n + 1}$$

4. (Continued) Determine whether each of the following series **converges** or **diverges**. Name any convergence test(s) you use, and justify all of your work.

(c) $\sum_{n=3}^{\infty} \left(-\frac{5}{6}\right)^n + \left(\frac{7}{8}\right)^n$

(d) $\sum_{n=1}^{\infty} \frac{e^{3n} n!}{n^n}$

5. [20 Points] In each case determine whether the given series is **absolutely convergent**, **conditionally convergent**, or **diverges**. Name any convergence test(s) you use, and justify all of your work.

(a)
$$\sum_{n=1}^{\infty} (-1)^n \frac{\arctan n}{7^n + 1}$$

(b)
$$\sum_{n=2}^{\infty} (-1)^n \frac{1}{n^3 - 1}$$

5. (Continued) In each case determine whether the given series is **absolutely convergent**, **conditionally convergent**, or **diverges**. Name any convergence test(s) you use, and justify all of your work.

(c) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n+7}$

OPTIONAL BONUS

Do not attempt these unless you are completely done with the rest of the exam.

OPTIONAL BONUS #1 Compute the sum of the following series:

1.
$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n(n+1)} + n\sqrt{n+1}}$$

OPTIONAL BONUS #2 Compute the following integral:

2.
$$\int_{-\infty}^{\infty} \frac{1}{x^{\frac{2}{3}}(1+x^{\frac{2}{3}})} dx$$

OPTIONAL BONUS

Do not attempt these unless you are completely done with the rest of the exam.

OPTIONAL BONUS #3 Compute the following integral:

3. $\int \frac{e^{2x}}{e^{8x} - 1} dx$

OPTIONAL BONUS #4 Compute the following integral:

4. $\int \frac{x^5 + 7x^3 + x^2 + 13x + 2}{x^4 + 6x^2 + 9} dx$