Math 121Midterm Exam #1March 11-14, 2021Due Sunday, March 14, in Gradescope by 11:59 pm ET

• This is an *Open Notes* Exam. You can use course materials, homeworks problems, lecture notes, etc. that you manually worked on.

• There is **NO** Open Internet allowed. You can only access our Main Course Webpage.

• You are **NOT** allowed to work on or discuss these problems with other people, including the Professor or Math Fellow TA.

- Submit your final work in Gradescope in the Exam 1 entry.
- Please show all of your work and justify all of your answers. No Calculators.

Limits [30 Points total, 10 points per each limit] Evaluate each of the following. Please justify/simplify. Be clear if the limit equals a value, $+\infty$ or $-\infty$, or Does Not Exist.

- **1.** Show that $\lim_{x \to 1} \frac{\arctan(x-1) \cos(\pi x) x}{\sinh(x^2 1) 2\ln x} = \boxed{-\frac{\pi^2}{4}}$
- **2.** Show that $\lim_{x\to 0^+} x^5 \ln x = \boxed{0}$

3. Show that
$$\lim_{x \to \infty} \left(1 - \arcsin\left(\frac{2}{x^4}\right) \right)^{x^4} = \boxed{e^{-2}}$$

Integrals [70 Points total, 10 points per each integral]

4. Compute
$$\int_1^3 \frac{1}{\sqrt{x} \cdot \sqrt{4-x}} dx$$
 Simplify.

5. Compute
$$\int_{e}^{e^3} \frac{1}{x [3 + (\ln x)^2]} dx$$
 Simplify

6. Show that $\int_{-2}^{2} \sqrt{4-x^2} \, dx = \boxed{2\pi}$ using Trigonometric Substitution

- **7.** Compute $\int \frac{x}{\sqrt{4+x^2}} dx$ using Trigonometric Substitution
- 8. Compute $\int x^2 \arcsin x \, dx$

9. Show that
$$\int_{1}^{e} [\ln(x^{3})]^{2} dx = 9e - 18$$

10. Compute
$$\int \frac{e^{2x}}{(4+e^{4x})^{\frac{3}{2}}} dx$$
 Hint: $e^{4x} = (e^{2x})^2$