Math 121   Midterm Exam #1   February 21, 2020

• This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or other aids are permitted.

• You need *not* simplify algebraically complicated answers. However, numerical answers such as $\sin\left(\frac{\pi}{6}\right)$, $4^{\frac{3}{2}}$, $\sinh(\ln 3)$, $e^{\ln 4}$, $\ln(e^7)$, or $e^{3\ln 3}$ should be simplified.

• Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

1. [14 Points]

(a) Use implicit differentiation to **PROVE** that $\dfrac{d}{dx}\arctan x = \dfrac{1}{1+x^2}$.

(b) From part (a) we now know that $\displaystyle\int \frac{1}{1+x^2}\,dx = \arctan x + C$. You may use this fact to **PROVE** that

$$\int \frac{1}{3+x^2}\,dx = \frac{1}{\sqrt{3}}\arctan\left(\frac{x}{\sqrt{3}}\right) + C \qquad \longleftarrow \textbf{Prove this.}$$

2. [24 Points] Evaluate the following **limit**. Please justify your answer. Be clear if the limit equals a value, $+\infty$ or $-\infty$, or Does Not Exist. Simplify.

(a) $\displaystyle\lim_{x\to 0} \frac{\arcsin x + x^2 + \ln(1-x)}{\cosh(2x) - \arctan(5x) - e^{-5x}}$

(b) $\displaystyle\lim_{x\to\infty} \left[1 - \arcsin\left(\frac{5}{x^2}\right)\right]^{x^2}$

3. [42 Points] Compute each of the following **integrals.** Please simplify your answer.

(a) $\displaystyle\int_2^{2\sqrt{3}} \frac{1}{\sqrt{16-x^2}}\,dx$

(b) $\displaystyle\int \frac{x^2}{\sqrt{16-x^2}}\,dx$

(c) $\displaystyle\int_{\frac{\pi}{2}}^{\pi} \frac{\cos x}{3+\sin^2 x}\,dx$

(d) $\displaystyle\int \frac{1}{x\,[3+(\ln x)^2]^{\frac{3}{2}}}\,dx$

4. [20 Points] Compute each of the following **integrals**. Please simplify your answer.

(a) Show that $\displaystyle\int_0^1 x^2 \arcsin x \; dx = \frac{\pi}{6} - \frac{2}{9}$

(b) Show that $\displaystyle\int_0^1 (x+1) \arctan x \; dx = \frac{\pi - 1 - \ln 2}{2}$

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OPTIONAL BONUS

Do not attempt this unless you are completely done with the rest of the exam.

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OPTIONAL BONUS #1 Compute the following **indefinite integral**.

1. $\displaystyle\int x \sin^4 x \; dx$