

February 23, 2016

- This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or other aids are permitted.
- You need *not* simplify algebraically complicated answers. However, numerical answers such as $\sin\left(\frac{\pi}{6}\right)$, $\arcsin\left(\frac{1}{2}\right)$, $4^{\frac{3}{2}}$, $\sinh(\ln 3)$, $e^{\ln 4}$, $\ln(e^7)$, or $e^{3\ln 3}$ should be simplified.
- Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

1. [10 Points] Use implicit differentiation to **PROVE** that $\frac{d}{dx} \sinh^{-1} x = \frac{1}{\sqrt{1+x^2}}$.

2. [30 Points] Evaluate each of the following **limits**. Please justify your answers. Be clear if the limit equals a value, $+\infty$ or $-\infty$, or Does Not Exist.

(a) $\lim_{x \rightarrow \ln 2} \frac{8 - e^{3x}}{e^{-4x} - \frac{1}{16}}$

(b) $\lim_{x \rightarrow 0} \frac{\sin^{-1} x + \cos(3x) - e^x}{\sinh^{-1} x + x^2 - \sinh x}$

(c) $\lim_{x \rightarrow \infty} \left(1 - \arctan\left(\frac{5}{x^2}\right)\right)^{x^2}$

3. [45 Points] Compute the following **definite integral**. Please simplify your answer.

(a) $\int_0^{\sqrt{3}} x \arctan x \, dx$

(b) $\int_1^{\sqrt{3}} \frac{x^2}{\sqrt{4-x^2}} \, dx$

(c) $\int_1^3 \frac{1}{\sqrt{x}(x+3)} \, dx$

(d) $\int_1^{\sqrt{e}} [\ln(x^2)]^2 \, dx$

4. [15 Points] Compute the following **indefinite integral**.

$$\int \frac{\cos x}{(4 + \sin^2 x)^{\frac{5}{2}}} dx$$

OPTIONAL BONUS

Do not attempt this unless you are completely done with the rest of the exam.

OPTIONAL BONUS #1 Compute the following **indefinite integral**.

1. $\int \frac{x^4 - 8x^3 + 24x^2 - 32x + 16}{(4x - x^2)^{\frac{7}{2}}} dx$