Math 121   Midterm Exam #1   February 20, 2015

• This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or other aids are permitted.

• You need *not* simplify algebraically complicated answers. However, numerical answers such as $\sin\left(\dfrac{\pi}{6}\right)$, $4^{\frac{3}{2}}$, $\sinh(\ln 3)$, $e^{\ln 4}$, $\ln(e^7)$, or $e^{3\ln 3}$ should be simplified.

• Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

1. [10 Points]

(a) Use implicit differentiation to **PROVE** that $\dfrac{d}{dx}\sinh^{-1}x = \dfrac{1}{\sqrt{1+x^2}}$.

(b) From part (a) we now know that $\displaystyle\int \dfrac{1}{\sqrt{1+x^2}}\,dx = \sinh^{-1}x + C$. Use this fact **and integration** to **PROVE** that

$$\int \frac{1}{\sqrt{3+x^2}}\,dx = \sinh^{-1}\left(\frac{x}{\sqrt{3}}\right) + C \quad \longleftarrow \textbf{Prove this.}$$

2. [30 Points]   Evaluate each of the following **limits**. Please justify your answers. Be clear if the limit equals a value, $+\infty$ or $-\infty$, or Does Not Exist.

(a) $\displaystyle\lim_{x\to 0}\ \frac{1 - e^{-3x} - \arctan(3x)}{x^2}$

(b) $\displaystyle\lim_{x\to\infty}\ \left(1 - \frac{2}{x^3}\right)^{7x^3}$

(c) $\displaystyle\lim_{x\to\infty}\ (x + \ln x)^{\frac{3}{x}}$

3. [30 Points]   Compute the following **definite integrals**. Please simplify your answer.

(a) $\displaystyle\int_0^{\ln 3} \frac{x}{e^x}\,dx$

(b) $\displaystyle\int_0^1 \ln(x^2 + 1)\,dx$

(c) $\displaystyle\int_4^{4\sqrt{3}} \frac{1}{\sqrt{64 - x^2}} + \frac{1}{16 + x^2}\,dx$

4. [30 Points]   Compute the following **indefinite integrals**.

(a) $\displaystyle\int \frac{1}{\sqrt{1-x^2}\,[1+(\arcsin x)^2]}\, dx$

(b) $\displaystyle\int x \arcsin x \, dx$

(c) $\displaystyle\int \frac{1}{x\,[9+(\ln x)^2]^{\frac{5}{2}}}\, dx$

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OPTIONAL BONUS

Do not attempt this unless you are completely done with the rest of the exam.

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OPTIONAL BONUS #1   Compute the following **indefinite integral**.

1. $\displaystyle\int \frac{xe^x}{\sqrt{1+e^x}}\, dx$