

- This is a closed-book examination. No books, notes, calculators, cell phones, webpages, communication devices of any sort, or other aids are permitted.
- You need *not* simplify algebraically complicated answers. However, numerical answers such as  $\sin\left(\frac{\pi}{6}\right)$ ,  $4^{\frac{3}{2}}$ ,  $\sinh(\ln 3)$ ,  $e^{\ln 4}$ ,  $\ln(e^7)$ , or  $e^{3\ln 3}$  should be simplified.
- Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

1. [6 Points] Let  $y = \arcsin x$ . Use implicit differentiation to **PROVE** that  $\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$ .

2. [24 Points] Evaluate each of the following **limits**. Please justify your answers. Be clear if the limit equals a value,  $+\infty$  or  $-\infty$ , or Does Not Exist.

(a)  $\lim_{x \rightarrow 0} \frac{\cosh(4x) - 1 - \arctan(4x) + 4x}{\ln(1-x) + x}$

(b)  $\lim_{x \rightarrow \infty} \left(1 - \frac{3}{x}\right)^{5x}$

(c)  $\lim_{x \rightarrow \infty} (\ln(x^2 + 1))^{\frac{1}{x^2}}$

3. [30 Points] Compute the following **definite integral**. Please simplify your answer.

(a)  $\int_0^{\ln 5} \sinh x \, dx$

(b)  $\int_2^{2\sqrt{3}} \frac{1}{\sqrt{16-x^2}} + \frac{1}{4+x^2} \, dx$

(c)  $\int_1^{\sqrt{e}} \ln(x^2) \, dx$

4. [40 Points] Compute the following **indefinite integral**.

(a)  $\int x \arcsin x \, dx$

(b)  $\int \frac{1}{(x^2 + 4)^{\frac{7}{2}}} \, dx$

(c)  $\int \frac{e^x}{1 + e^{2x}} \, dx$

(d)  $\int \frac{x^2}{e^{5x}} \, dx$

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## OPTIONAL BONUS

Do not attempt these unless you are completely done with the rest of the exam.

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**OPTIONAL BONUS #1** Compute the following **indefinite integral**.

1.  $\int \frac{x^3}{1 - \sin(x^2)} \, dx$

**OPTIONAL BONUS #2** Compute the following **indefinite integral**.

2.  $\int \frac{1}{1 + 3 \sin^2 \theta} \, d\theta$